STANDARD ARITHMETIC No. 2.

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Fl. Cajori

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PUBLISHER'S NOTICE.

THIS sample copy of STANDARD ARITHMETIC No. 2 is sent out only for examination and correction. The first regular edition will have a Key appended similar to No. 1. The Arithmetic is now undergoing critical revision by competent hands, and in the forthcoming edition all errors will be corrected.

THE

STANDARD ARITHMETIC,

FOR

SCHOOLS OF ALL GRADES

AND FOR

BUSINESS PURPOSES.

In two Numbers.

NUMBER TWO.

BY

JAMES E. RYAN,

PRINCIPAL OF SCHOOL NO. 26, BROOKLYN.

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CONTENTS.

	PAGE
Preface,	5
Suggestions to Teachers,	7
Introduction,	9
Notation and Numeration,	9
Notation Principles,	11
Addition,	13
" Axioms of,	13
" Problem Table,	15
Subtraction,	16
Multiplication,	17
Division,	18
Forms of Bills,	23
Signs and Abbreviations,	24
Properties of Numbers,	25
Methods of Proof,	27
Greatest Common Divisor,	29
Least Common Multiple,	30
Questions for Gen. Review,	31
Problems for Gen. Review,	32
Common Fractions,	34
Reduction of Com. Fractions,	36
" Problems,	37
Addition of Fractions,	38
Subtraction of Fractions,	39
Multiplication of Fractions,	39
Division of Fractions,	40
Statement of Problems,	41
Decimal Fractions,	42
General Principles of D. F.,	43
Notation of Decimals,	44
Reduct'n of Com. F. to Dec.,	46
Repetends,	47
Miscell. Problems in Dec.,	49
Denominate Numbers,	51

	PAGE
Tables of Money,	51
" of Weights,	52
" of Lengths, Area, etc.,	53
Table of Germ. Currency,	54
Reduction of Den. Numbers,	55
Fractional Den. Numbers,	58
Decimal Den. Numbers,	59
Percentage,	61
Interest,	65
" at Sixty Days,	66
" General Rule,	67
" Aliquot Parts,	68
" Accurate N. Y. Law,	70
Principal, Rate, or Time,	71
Bank Discount,	74
Proceeds of a Note,	76
Face of a Note,	77
True Discount,	77
Commercial Discount,	79
Interest Notes,	80
Compound Interest,	81
Partial Payments,	83
Exchange,	85
Sterling Exchange,	88
French "	89
German "	90
Income from Stocks,	91
Rentals,	93
Profit and Loss,	93
Commission or Brokerage,	95
Insurance,	97
Taxes,	98
Duties,	99
Equation of Payments,	100

	PAGE
Averaging Accounts,	102
Accounts with Cash Balance,	105
Ratio,	106
Simple Proportion,	107
Direct Ratios,	111
Inverse Ratios,	114
Compound Proportion,	114
Analysis,	117
Involution,	122
Square Root,	124
Cube Root,	128
Questions on Roots, etc.,	134
Mensuration,	135
Areas, etc., of Triangles,	137
Areas of Polygons,	140
The Circle,	143
The Square and Circle,	148
Surface of Spheres,	148
" Prisms,	149
" Rings,	149
" Cones,	150
Volume,	152
" Parallelopipeds,	152
" Prisms,	152
" Polyhedrons,	152
" Cylinders,	153
" Cones, etc.,	153
Problems in Mensuration,	154
Irregular Bodies,	158
Arithmetical Progression,	159
Geometrical Progression,	161
Repetends,	163
Appendix,	167
Signs and Letters,	167
Equations,	168
Falling Bodies,	170
Machinery,	171
Tensile Strength.	172
Transverse Strength,	173
Specific Gravity,	174
Builders' Measurements,	175
Doyle's Rule (Lumber),	176
Gauging of Casks,	176
Alligation,	176
Life Insurance,	177
Compound Interest Table,	179
Expectation of Life,	180
Annuities,	181
Legal Interest and Usury,	184
Table of Days,	185
Stocks and Bonds,	185
Stockbrokers' Phrases,	186
Custom-House Business,	187
Value of Foreign Coins,	188
Freight and Storage,	188
R.R. Freights, Table,	189
Remarks on Tables,	190
Miscellaneous Problems,	191
Vocabulary,	195

PREFACE.

An Arithmetic constructed on a new plan and filled with novel theories and curious problems might serve as a means of mental discipline, but would utterly fail to train youths for business. The precepts and processes representing the accumulated wisdom of the past even Newton did not ignore. There are ponderous treatises on Arithmetic which comprise nearly every principle and formula pertaining to the science. These are for the most part very abstruse; and are, without exception, bulky and expensive.

It has been the aim of the author to make this text-book equal to the most costly as respects facts, principles, processes, and problems, and to make the chapters in general use, such as those on Notation, Denominate Numbers, Common and Decimal Fractions, and Mensuration, even more interesting and more practical than those of the larger volumes; in fine, to furnish a convenient, complete, and cheap manual of arithmetic.

For the sake of convenience and economy the work is published in two parts.

PART FIRST is chiefly devoted to the discussion of the "Ground Rules," Common and Decimal Fractions, and Denominate Numbers. It contains, also, a carefully-written chapter on the Measurement of the simpler forms of Surfaces and Solids, and a brief but comprehensive chapter on Percentage and its Applications. This was prepared to meet the requirements of all "grades" below the "first" in our Schools.

The opening chapters of PART SECOND are equivalent to PART FIRST. They will be found sufficient for the instruction of most beginners, as well as convenient for purposes of reference and review.

The data which form the basis of the numerous problems relating to commerce and the arts are from reliable sources. These constitute a fund of valuable information, and, by making the conditions of each problem clear and life-like, tend to simplify the arithmetical processes, and to familiarize learners with actual business transactions. Complex "synopses" have been purposely omitted. No small type has been used for the reason that the work contains nothing which may be safely disregarded. Some slight changes in nomenclature have been made with a view to brevity and clearness. The Appendix contains valuable Tables, Specimen Examination Papers, and Explanations of Commercial Terms.

BROOKLYN, N. Y., June 1, 1877.

SUGGESTIONS TO TEACHERS.

MAKE sure that every definition and principle is clearly comprehended, and then thoroughly memorized. Direct attention to the intimate relations of the different operations, and to the fact that the principles on which the various rules are founded are very few and very simple.

Study how best to expand condensed statements and to furnish apposite illustrations.

Do not give direct aid; rather suggest such easier problems as will lead the learner gently upward, and make every failure an occasion for searching questions on what has been done previously, in order to secure a clearer comprehension of the principles involved.

Drill pupils in the art of *stating the work to be done* before they begin to perform the fundamental operations required. Do not take it for granted because you can state all the processes in regular sequence, that the matter is equally clear to all the members of a class. After the solution require your pupils *to state plainly what has been done*, and, above all, demand that they shall PROVE THEIR WORK.

BE EXPLICIT. Questions should be plain and fair. In assigning lessons, let there be no doubt as to what is to be learned.

BE WATCHFUL. Be satisfied with nothing less than perfect attention.

The body of every lesson should be recited by the feebler pupils, while the more clever members of the class should be kept alert as auxiliaries and referees.

Pupils should be prompt and concise. They should be made familiar with ordinary technical words. Every craftsman must get familiar with his tools. Such terms as factor, exponent, multiple, L. C. D., L. C. M., per cent., area, angle, and cube are as necessary to clear conceptions as sum, quotient, or remainder, and should be employed instead of their more extended equivalents.

In conducting slate exercises be BRIEF. Proceed somewhat as follows: "Slates." —— Teacher reads problem.—"Ready." Teacher reads problem again, *slowly*, while the pupils note the data. —— "Time." Pupils cease working.—"Exchange slates and criticise in writing." —— "Return slates." "Excuses." Class called by *numbers* and credits given. "Blackboard solution by Smith." Smith solves and explains. "Class criticise." —— Drill of backward pupils.

(Directions by teacher in quotation marks. An allowance of time shown by a dash.)

INTRODUCTION.

1. ***Arithmetic*** treats of the properties and relations of **Number.**

2. A **Number** consists of one or more units, and answers the question *How many?*

3. A **Unit** is anything considered as a whole—that is, without reference to its parts—as a score, a hundred.

4. **The Primary Unit** is a single thing or one.

5. A number used without reference to any object is a **pure** or **abstract number**; as five, two-thirds.

6. A **concrete number** refers to some object named; as *six* in the phrase "six cents."

7. **Notation** is the art of writing numbers in characters.

8. **Numeration** is the art of expressing numbers in words when they have already been written in figures.

9. The **Arabic system of notation** is used in Arithmetical computations. Sometimes **letters** are used to stand for numbers for the sake of convenience. Thus, *a* for 250,000.

10. The **ten symbols** or Arabic characters are

0,	1,	2,	3,	4,	5,	6,	7,	8,	9,
naught,	**one,**	**two,**	**three,**	**four,**	**five,**	**six,**	**seven,**	**eight,**	**nine,**

of which the last nine are called **digits,** or **significant figures.** The first is called *naught, zero,* or *cipher.*

11. **Each digit standing alone** represents one or more units.

12. When *more than nine* are to be expressed, two or more figures are required. These are placed side by side; that on the right counts for *units,* the next figure counts for

tens; the next, for tens of tens, or hundreds; the next, for tens of tens of tens, or thousands, etc.

13. A **ten** is a group of ones considered collectively as one.

Orders.

14. Numbers are made up of **orders.** The first order represents simple units, or ones; the second order stands for tens; the third order, for hundreds; the fourth order, for thousands.

15. Orders increase from right to left **by the scale of ten.** See the following

Table.

Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
6	6	6	6	6	6	6	6	6	6	6

16. A **cipher** by itself expresses no value. It is used to mark vacant orders, and thus to locate other figures. *Safira* (Arabic) means to be empty.

Periods.

17. Large numbers are separated by commas into **periods of three figures** each. We then have *Units, Tens,* and *Hundreds* of each period. The names of the periods, or grades, counting from the right, are as follows: Units, Thousands, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, etc.

Periods, .	5th			4th			3d			2d			1st		
Orders, .	15,	14,	13,	12,	11,	10,	9,	8,	7,	6,	5,	4,	3,	2,	1
Names, .	TRILLIONS.			BILLIONS.			MILLIONS.			THOUSANDS.			UNITS.		
	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
Number,	2	2	2	2	2	0	0	2	2	2	0	2	2	2	2

This number is read: Two hundred and twenty-two trillions, two hundred and twenty billions, twenty-two millions, two hundred and two thousand, two hundred and twenty-two.

18. The word **units** is not usually uttered; but tens and hundred (tens abbreviated to *ty*) are always heard when represented by a digit.

19. Pupils should become quite ready in the application of the **names** and **orders** to at least 13 figures, so as to answer promptly such questions as these: What is the name of the tenth order? How many places are needed for 12 millions? How many for 9 trillions? For 1 billion?

Principles of Notation and Numeration.

20. I. EACH SEPARATE FIGURE ALWAYS EXPRESSES THE SAME NUMBER OF UNITS.

II. TEN OF ANY ORDER MAKE ONE OF THE NEXT HIGHER.

III. CIPHERS MARK ALL VACANT ORDERS.

The following exercises will serve for practice in Notation,

and subsequently for problems in Addition. Accurate notation is of the first importance; mistakes in other work can be readily found; errors in Notation can hardly be remedied, as the spoken words leave no trace behind.

Express in figures the following:

(1) Sixty-one; seventy-nine; nineteen; ninety-seven; eleven. (2) Three hundred; two hundred and three; four hundred and sixty-four; three hundred and fifty; one hundred. (3) Nine thousand; four thousand, four hundred and one; thirty; five thousand and five. (4) Thirty-eight thousand; one thousand and twenty; sixteen thousand and sixteen. (5) One hundred thousand, seven hundred and seven; seventy-seven thousand and seventy. (6) Nine hundred; two millions; sixty-five millions and sixty-five; four hundred and ten millions and eighty. (7) Seven billions, seven millions, seven thousand and seven; five hundred and four billions and sixteen. (8) Eighty-two trillions; one hundred trillions, one million, ten thousand and fifteen. (9) Sixty-four trillions, sixty-four thousand, four hundred and eighty-two. (10) Nine; ninety-one; seven thousand and eighty; four millions and five hundred.

Write in words at full length the following numbers:

1. 6; 60; 106; 900; 9001; 40004; 4000040; 70007.
2. 8001142; 9300466; 3462; 718; 10462; 184793.
3. 123456; 343434; 567890; 1357900; 215.
4. 1122004; 6824; 312590; 8235; 53367; 9102.
5. 3003003003000; 987002654; 3210964300.

Teachers may find it necessary to proceed somewhat slowly in some instances. Such numbers as those subjoined, with periods indicated by commas, may be treated thus: Read the units, then the thousands, then the millions, then the total.

412,325,124
683,121,459
186,245,820
. . . .

Three sets of dots will show the places of the periods and will assist learners in pointing out the required place of any figure when questioned: thus, he will touch the eighth dot if

thirty millions are pronounced; the sixth, if two hundred thousand be required.

21. A **Rule** shows how to do what is required.

22. A **Principle** is an essential part of a subject.

An **Axiom** is a simple statement of a self-evident truth.

A **Problem** proposes something to be done.

A **Solution** is equivalent to a rule, and should fully exemplify the steps of the process.

23. ***Axiom I.*** **All the parts of a number make up the whole number.** $100+20+3$ equal 123.

24. ***Axiom II.*** **If we add all the parts of several numbers, we add the numbers.**

25. ***Axiom III.*** **The similar orders of abstract numbers must be collected by themselves;** thus, units must be joined to units, tens to tens, etc., and **concrete numbers to be added must be of the same nature.** Bushels, gallons, and cubic feet, *when reduced,* may be added, as they all express capacity, **and have a common unit:** viz., the cubic inch. All weights and measures may be referred to the metric system (Tables, Art. 147) as a means of comparison.

26. ***Addition*** is the process of finding a number which will exactly equal two or more other numbers when taken together.

27. The numbers to be added are called addends. They are usually written one above another with like orders in the same column.

28. The number found by addition is called a sum.

29. The sign of addition ($+$) is called plus.

30. A problem in addition may be expressed thus: $4+3+2=?$; read thus: 4 plus 3 plus 2 equals what?

31. Two parallel horizontal lines make the sign of equality, $=$. It is read equal, equals, or is equal to.

32. When either of two names will exactly express the value of a concrete number—as, two tens, twenty units or a score, four pecks or one bushel—one may stand for the other.

33. **Reduction** is the substitution of one number for another of equal value, It consists in a change of *names*.

34. ***Axiom IV.*** **A lower order having a sufficient number of units may be reduced to a higher order.**

35. ***Axiom V.*** **The value of a quantity is not altered if what is taken from one part is added to another.**

NOTE.—It is taken for granted that pupils using this book have learned the addition table, or that they can easily find or make one. No pupil is likely to be so thorough as not to derive decided advantage from *practice* in addition.

36. The class should be required to give a thorough analysis of each of the three following problems, and then to practise, frequently, exercises based on Tables A and B.

3	4	5	8
2	0	6	1
3	4	9	7
8	2	3	4
16	10	23	20
17	2	5	0

8	4	1	9	2
7	6	2	3	5
8	4	2	7	6
9	9	9	9	9
32	23	14	28	22
34	4	7	0	2

5	8	2	6
3	9	7	0
9	8	2	8
4	6	3	9
21	31	14	23
24	2	6	3

a	*b*	*c*
3	5	7 (72)
7	7	9 (66)
6	6	6 (56)
4	5	7
8	4	8 (46)
2	3	9 (36)
9	2	4 (26)
1	9	6
5	1	8 (16)
5	8	8
50	50	72

Addition by tens. Add column *a* thus: 10, 20, 30, 40, 50. Collect the scattered pairs of column *b* thus: (5+5), (7+3), (6+4), (2+8), and (9+1); pronounced 10, 20, 30, 40, 50. If added upward, the work would be pronounced "9, 18, 27, 37, 47, 50." *For the instant,* 6+5 and 7+5 are each added as 10, the procession of 7s going on mechanically, the memory noting the excesses, 1 and 2, which are added to 47 at the end. In column *c*, pronounce thus: 16, 26, 36, 46, 56, 66, 72. The deficiency arising from counting 8 and 9 as tens, (36), (46), is covered by the excess (3) in the next two numbers (7+6).

TABLE A.		TABLE B.	
1 0 1 1 3 2 5	*x*	4 0 4 4 0 4 3 0 3	*x*
4 2 8 3 5 7 4 9	*w*	2 2 0 0 2 2 0 0 2 2	*w*
7 1 9 3 6 0 6	*v*	9 1 7 9 6 7 9 4 7	*v*
9 4 7 9 3 5	*u*	3 8 4 3 8 8 5 6	*u*
9 8 3 6 4 2 8 2	*t*	1 4 2 3 9 1 8 2 5 4 6	*t*
2 7 3 5 4 4 9 1	*s*	1 9 4 8 5	*s*
1 2 4 0 0 6 5 3	*r*	5 8 9 2 8 4 6	*r*
6 7 5 2 8 3 5 4 8	*q*	1 5 2 4 2 8 3 4	*q*
9 5 4 8 7 9 2 4 3	*p*	8 9 3 6 1 4 8 3	*p*
1 4 0 6 6 7 9 2 2 8	*o*	6 4 2 8.7 8 9 2 4	*o*
9 8 1 2 3 4 5 6 7 3	*n*	3 3 2 8 1 4 2 4 9 3	*n*
8 7 6 5 4 3 2 1 9 8	*m*	1 4 5 9 6 8 4	*m*
1 3 4 2 8 7 6 5 4 3	*l*	2 8 3 6 7 9 5 9 5	*l*
3 2 8 2 8 3 7 6 5 4	*k*	8 3 3 6 7 4 9 4 5	*k*
3 6 1 5 4	*j*	1 2 4 7 7 8 8 6 6 6	*j*
4 0 4 0 4 0 4	*i*	5 1 8 4 0 0 4 7	*i*
1 8 2 4 0 6 7 5 3	*h*	8 3 7 5 0 4 9 8 3	*h*
2 4 0 0 6 4 0 5	*g*	5 8 2 7 6 3	*g*
8 1 9 3	*f*	9 4 5 8 2 1 4 6	*f*
7 2 4 6 5 9	*e*	4 1 1 2 8 9 3 9 3 9 3	*e*
1 9 0 0 0 4 8	*d*	5 9 7 6 8 2 4 5 4	*d*
8 1 9 2 3 7 5 0 4 4	*c*	5 4 2 0 0 4 7 9 6 3 0	*c*
5 1 7 4 2 6	*b*	1 2 4 3 6 4 5 2 8 5 4	*b*
6 4 3 9 2 8 7	*a*	4 9 3 6 8 5 2 4 2 4	*a*

The numbers written under A and B can be combined in many ways, and will afford scores of problems to an ingenious teacher.

The questions which follow are merely introductory and suggestive.

37. Note.—These numbers may be used for problems in Numeration, Subtraction, Multiplication, Division, Fractions, Denominate Numbers, Square Root, Cube Root, Mensuration, etc. Thus: "Find the number of inches in *b* furlongs, Table A"; in *j* yards; etc.

EXERCISE I. A.	EXERCISE II. B.
1. Req. the sum of the Nos. above *q*.	23. Ditto.
2. " " " " *j*.	24. "
3. " " " " *f*.	25. "
4. " " " " *c*.	26. "
5. " " " " *b*.	27. "
6. " " " " *s*.	28. "
7. " " " " *l*.	29. "
8. " " " " *n*.	30. "
9. " " " " *v*.	31. "
10. Req. sum of *all* the Nos. under A.	32. "
11. " " *i j k l m* and *n*.	33. "
12. " " *a b c d e f g* and *i*.	34. "
13. " " *q r s t u v* and *w*.	35. "
14. " " *a c e g i k* and *m*.	36. "
15. " " Nos. between *a* and *q*.	37. "
16. " " including *k* and *x*.	38. "
17. " " " *g* and *r*.	39. "
18. " " " *h* and *n*.	40. "
19. " " *a d g j m p s* and *v*.	41. "
20. " " *b e h k n q t* and *w*.	42. "
21. Count from *c* adding every 3d No.	43. "
22. " " *b* " " 5th No.	44. "

NOTE.—The questions under Table A may be applied to Table B, and, when so applied, are numbered 23, 24, 25, etc.

38. ***Subtraction*** is the process of finding the **difference** between two numbers. The larger number is called the **minuend**; the smaller, the **subtrahend**; the difference is sometimes called the **remainder.**

39. Subtraction is indicated by the sign —, called minus, written after the larger number, and before the smaller one.

40. As the difference added to the subtrahend equals the minuend, the principles and axioms of Addition and Subtraction are alike. Besides these, it may be desirable to refer to

41. ***Axiom VI.*** **We do not alter the difference between two numbers if we add an equal number to each.**

NOTE.—Practice by *equal additions,* as well as by the more modern method of diminishing the preceding figures of the minuend. (See *Part First.*)

42. ***Multiplication.*** **Multiplication** is the process of repeating one number as many times as there are units in another. The number repeated is called the **multiplicand.**

The number which shows how many times the multiplicand is to be repeated, or added, is called the **Multiplier.**

43. The result is called the **Product.** The multiplier and multiplicand are called **Factors** of the *product.*

44. The **sign of multiplication** is an oblique cross, $\times$. It is placed between the factors.

45. A problem in multiplication is written thus: $2\times5=10$; and is read thus: 2 multiplied by 5 equals 10.

NOTE.—In *theory,* the multiplier must be **abstract.** In *practice,* either number may be used as a multiplier.

46. Multiplication is a **short method of addition** in which each of the addends equals the multiplicand taken once, the sum being called a product.

NOTE.—It seems unnecessary to print the Multiplication Table. Use the following as an oral exercise to show that addition and multiplication are alike:

10	20	30	40	50	60	70	80	90
9	18	27	36	45	54	63	72	81
8	16	24	32	40	48	56	64	72
7	14	21	28	35	42	49	56	63
6	12	18	24	30	36	42	48	54
5	10	15	20	25	30	35	40	45
4	8	12	16	20	24	28	32	36
3	6	9	12	15	18	21	24	27
2	4	6	8	10	12	14	16	18
1	2	3	4	5	6	7	8	9

47. The axioms applicable to addition are, for the most part, useful in explaining multiplication.

48. ***Axiom VII.*** **We multiply a number by another when we multiply every part of the one by every part of the other and add the results.**

49. The numbers and signs included in a **parenthesis** are to be considered as **an entire quantity.**

50. The product of two or more factors (each integral and greater than 1) may be called a **composite number.**

To Multiply by a Composite Number.

51. Rule.—*First multiply the multiplicand by one factor of the multiplier, then multiply the result by the next factor, and continue till all have been employed as multipliers.*

$1234 \times 84 = 1234 \times 3 \times 4 \times 7$; or, (3702), (14808), 103656.

52. A naught written at the right of a line of figures locates each one of the figures in its next higher order. Hence, annexing a naught multiplies the whole number by 10. Annexing two naughts to a number makes the number thus formed 100 times as large as before. Thus : 12=12 units ; 1200=12 hundreds. (For problems see page 15.)

53. ***Division teaches,***

I. The process of finding the number of times that one number is contained in another.

II. The process of dividing a number into equal parts.

III. The process of finding a factor when a product and the other factor are given.

54. The number to be divided is called the **Dividend ;** that which divides it is called the **Divisor.** The number which shows how often the dividend contains the divisor is called the **Quotient.** The number left over is called the **Remainder.**

55. The **Sign of division** is a short horizontal line between two dots. Thus, $12 \div 3$ is read: 12 divided by 3. 12 is the dividend, 3 is the divisor, and 4 is the quotient. The dividend should be written *before* the sign, and the divisor after it.

56. Division is often shown by writing a divisor under a dividend, with a line between. Thus, $\frac{14}{2}=7$ is read 14 divided by 2 equals 7.

57. A problem in division may be stated in various forms. The division of 21 by 7 is required by each of the following:

a. Divide 21 by 7.
b. How many times is 7 contained in 21?
c. What number multiplied by 7 will produce 21?
d. What is one of the 7 equal parts of 21?
e. If 7 is one factor of 21, what is the other?
f. Divide 21 into as many parts as there are units in 7.
g. How many times can 7 be subtracted from 21?

58. Note.—Multiplication is the reverse of Division. (*See c and e.*) Division is but a short method of Subtraction. (*Vide g.*) To divide 21 by 7 is but to take 7 three times from 21; thus: $21-7-7-7=14,\ 7,\ 0$. In training pupils to use the multiplication table, for purposes of division, at first formulate the questions thus: 4 times 5 are what? 3 times 7 are how many? Then alternate with such questions as these: 4 times what are 20? 3 times what are 21?

Divide 225 by 5.

Solution 1.			Solution 2.
5)200+20+5	or	5)225	6)756
40+ 4+1=45		45	126

Exercise III.

59. Find the 42 quotients required by the following: Divide each of these 6 dividends, viz., 116942, 700415, 30405060, 7194200, 842900, 7500004, by each of these 7 divisors, viz., 4, 5, 6, 7, 9, 11, and 12.

Correspondence of Multiplication and Division.

```
         3087
          456
        -----
        18522 =   6×3087
       15435  =  50×3087
      12348   = 400×3087
3087)1407672(   456×3087
      12348
      -----
       17287
       15435
       -----
        18522
        18522
        -----
```

The process of division is plainly the opposite, or "undoing," of multiplication. We have to find out the numbers 12348, 15435, and 18522, one by one, and to subtract these parts from the dividend.

Before learners are allowed to pass on from this subject, each pupil should be required to manifest his ability to analyze work like this example. When a blackboard problem is in the process of solution, as each quotient figure is brought down its value should be stated.

Rule.—*Place the divisor, dividend, and quotient thus:*

Divisor) Dividend (Quotient

Take of the left-hand figures of the dividend as many as will contain the divisor at least once and not more than nine times, and find how often the first left-hand figure of the divisor is contained in the first one or two left-hand figures of the dividend; the result must be written as the left-hand figure of the quotient. Multiply the divisor by this figure, and subtract the product from the figures first taken.

To the remainder annex the next dividend figure for a new partial dividend and proceed as before. If the divisor be greater than this number, affix a cipher to the quotient, and annex another dividend figure to the partial dividend. Thus proceed until all the figures of the dividend have been brought down. Write the last remainder, if there be any, over the divisor and consider it as a part of the quotient.

Proof.—Multiply the integral part of the quotient by the divisor and add in the remainder. The sum should equal the dividend. (See Art. 84.)

60. Teachers should not fail to induce their pupils to consider the *close correspondence of Division and Multiplication.* In every case of Division, the Divisor and Quotient are *Factors* of the Dividend.

61. Divisor $\times$ quotient $=$ dividend; multiplicand $\times$ multiplier $=$ product. Hence the principles, or Axioms, of Multiplication and Division are practically the same.

62. I. Multiplying the divisor, or dividing the dividend, divides the quotient.

II. Multiplying the dividend, or dividing the divisor, multiplies the quotient.

III. Multiplying both dividend and divisor leaves the quotient unaltered.

IV. Dividing both dividend and divisor will not alter the quotient.

I. Illustrated: $\frac{24}{6}=4$; $\frac{24}{6\times2}=2$; $4\div2=2$; so also, $\frac{24\div2}{6}=2$

II. Illustrated: $\frac{24}{6}=4$; $\frac{24\times2}{6}=8$; $4\times2=8$; so also, $\frac{24}{6\div2}=8$

III. Illustrated: $\frac{24}{6}=4$; $\frac{24\times3}{6\times3}=\frac{72}{18}=4$

IV. Illustrated: $\frac{24}{6}=4$; $\frac{24\div3}{6\div3}=\frac{8}{2}=4$

63. ***Axiom VIII.*** **We divide by a number when we divide by its factors successively in such a mode that the quotient of the first shall be divided by the next factor, and its quotient by the next, and so on.**

Divide 944 by 105, using factors. *Ans.* $8\frac{104}{105}$.

$105=5\times3\times7$	5)944		
	3)188	$4=$ 1st rem.	4
	7) 62	$2=$ 2d rem.	$2\times5=10$
	8	$6=$ 3d rem.	$6\times3\times5=90$
$944\div105=8\frac{104}{105}$.		True rem.,	104

To find the True Remainder.

64. Rule.—*Multiply each remainder by all the preceding divisors except that from which it arose, and add these products together for the true remainder.*

Exercise IV.

Find the quotients of the following, using the factors of the several divisors:

1–5. $\frac{8217}{35}$ $\frac{9591}{(2\times4\times9)}$ $\frac{801462}{108}$ $\frac{987654}{63}$ $\frac{926541}{81}$

65. Cutting off a figure from the right of a number brings the tens into the units' place, the hundreds into the tens' place, thousands into hundreds' place, and so on, thus reducing each order by one. If a figure be removed from the right, the number affected is divided by 10; if two figures are cut off, it is divided by 100; if three figures, by 1,000. The figures cut off must be considered as a *remainder.*

To Divide by a Number having Naughts at its right.

66. Rule.—*From the right of the dividend cut off as many figures as there are naughts at the right of the divisor. Divide by the rest of the divisor. Prefix the remainder to the figures cut off, for the true remainder.*

Exercise V.

Perform the work indicated by the following:

1–3. $\frac{376549281}{360\times480}$ $\frac{12345678}{420000}$ $\frac{89453}{2700}$

When one Product of several Factors is to be divided by another Product of several Factors.

67. Rule.—*Strike out the factors common to both divi-*

dend and divisor, and seek the result with those which remain.

Example: $\dfrac{36\times49\times105}{24\times14\times21}$ Here 12 is a factor found in 36 and in 24; 7 is common to both 49 and 14; and 7 and 3 are both found in 105 and 21. These being struck out of the respective products, there remain $(3\times7\times5)\div(2\times2)=26\frac{1}{4}$. The problem, as performed, will appear thus:

$$\frac{\overset{\cancel{3}}{\cancel{36}}}{\underset{2}{\cancel{24}}}\times\frac{\overset{\cancel{7}}{\cancel{49}}}{\underset{2}{\cancel{14}}}\times\frac{105}{\underset{\cancel{3}}{\cancel{21}}}=\frac{105}{4}=26\tfrac{1}{4}.$$

EXERCISE VI.

1–3. $\dfrac{16\times4\times5\times7}{8\times2\times35}$ $\dfrac{4\times5\times3\times7\times8}{3\times5\times4\times14}$ $\dfrac{20\times12\times48}{24\times4\times10}$

68. **BILL 1.**

LOCKPORT, June 10, 1874.

PAUL MANSON

To GEO. CLAY, *Dr.*

Jan.	1	To 51 yds. Muslin,	@ $0.22	. . .		
"	6	" 2 doz. Cuffs,	@ 0.30	. . .		
		" 1 " Collars,	@ 0.50	. . .		
"	18	" 2 pairs Gaiters,	@ 2.75	. . .		
Feb.	4	" 12 Hdkfs.	@ 0.40	. . .		
"	12	" 2 prs. Boots,	@ 5.50	. . .		
Mar.	9	" 4 yds. Cassimere,	@ 3.25	. . .		
		Cr.		$		
Mar.	9	By cash, ...		$	12	00
		Balance due?		$		

69. BILL 2.

MILWAUKEE, Dec. 31, 1875.

MR. SAMUEL VALENTINE

Bought of EDWARD BUSH.

1875.				
Mar.	1	3 cases Doeskins, 30 pcs., 30 yds. ea.@$2.00		
May	10	2 " F'cy Tweed, 28 " 32 " @ 1.20		
June	5	4 " Satinet, 25 " 28 " @ 0.92		
Sept.	10	6 " Cassimere, 30 " 31 " @ 2.25		
Oct.	12	8 " Br'dcloth, 20 " 42 " @ 4.35		
Nov.	12	9 " Flannel, 40 " 30 " @ 0.34		
		$		
		Cr.		
June	15	By cash, ... $	4400	00
Aug.	20	"	1600	00
Sept.	4	"	3000	00
		Balance due, ... $		
		Received pay't, EDWARD BUSH.		

70. *Signs and Abbreviations.*

a. The sign $+$, *plus,* indicates addition.

b. The sign $=$, *equals,* is placed between equal numbers.

c. The sign $\overline{\quad}$, *vinculum,* placed over numbers, and

d. The sign ( ), *parenthesis,* enclosing numbers, denote that the numbers included are equally affected by whatsoever is outside of, or beyond, the parenthesis or vinculum. Thus $\overline{2+3}\times\overline{5-2}=(2+3)\times(5-2)=5\times3$.

e. The sign $\therefore$ signifies *therefore,* and precedes the conclusion of an explanation.

f. The sign —, *minus,* signifies that the second of two quantities is to be subtracted from the first, as $12-9=3$.

g. The sign $\sim$, *difference,* shows that a difference is to be

found between two numbers. The larger may be written on the right. Thus $8 \sim 12 = 4$.

h. The sign $\times$ signifies that the numbers between which it stands are to be multiplied.

i. The sign $\div$ signifies that the first of the two numbers between which it stands is to be divided by the second.

j. The influence of $\times$ or $\div$ does not extend beyond the signs $+$ or $-$. Thus, $6+4\div2+18-3+27\div3=6+$"2"$+18-3+$"9," and is not equivalent to $(6+4)\div2+(18-3+27)\div3$. This last expression $=5+14$.

k. For such abbreviations as bu., pk., lb., see the *Tables.*

NOTE.—Pupils should be drilled carefully on these.

EXERCISE VII.

Find the value of the following expressions:

1. $15\times37153-73474+67152\div4+47034\times2$.
2. $494871-94853+(45079-3177)-2\times(54312-3987)$.
3. $\overline{16+25+33-2}\div12+18-24+3\times(241-116)$.
4. $(846+47+96)\times25-20000\div25$.

PROPERTIES OF NUMBERS.

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72. Numbers are sometimes classified as Prime and Composite.

73. A **prime number** has no other **exact** divisor than itself and unity. All other numbers are **composite.**

74. Numbers are said to be **prime to one another** when they have no common divisor—*e. g.,* 4 and 21.

75. The **factors** of a number are such other numbers as when multiplied will produce it.

76. A **factor** is sometimes called a **divisor** or **measure,** because it will exactly divide some composite number.

77. The **prime factors** of a number are the prime numbers which produce it. 3, 5, and 7 are the prime factors of 105.

78. A **factor** is **common** to two or more numbers when it is a factor or measure of each of them.

79. The greatest common divisor, G. C. D., of two or more numbers is the greatest number which will exactly divide each of them. 12 is the G. C. D. of 24, 36, 48, and 60. It is their greatest common factor.

80. Every **even** number will exactly contain 2; any number will exactly contain 4 if 4 is a measure of its **last two figures;** 8 is a measure if it will measure the **last three figures;** 9 is a measure, if it will measure **the sum of the figures;** 5 is a measure of any number ending in 0 or 5; 7, 11, and 13 are measures of all numbers of four figures composed of two like digits separated by two ciphers—*e. g.*, 1001, 7007.

81. A square number never ends in 2, 3, 7, or 8.

82. An **exponent,** or index, is a small figure written at the right and above a number to show how many times the number is taken as a factor. Thus, in the expression $32=2^5$, 5 is an exponent. $288=2^5\times3^2$ or $2\times2\times2\times2\times2\times3\times3$.

A SHORT TABLE OF PRIME NUMBERS.

1	11	23	41	59	73	97	109
3	13	29	43	61	79	101	113
5	17	31	47	67	83	103	127
7	19	37	53	71	89	107	131

METHODS OF SECURING ACCURACY.

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83. Strict proof is the opposite process. Subtraction is proved by Addition, Multiplication by Division, and *vice versa.*

84. ***Casting out the 9s*** affords a rapid and reasonably satisfactory test. **If the sum of the digits of a number be divided by 9, the remainder will equal that obtained by dividing the number itself by 9.**

$$\frac{158246}{9} = \frac{100000}{9} + \frac{50000}{9} + \frac{8000}{9} + \frac{200}{9} + \frac{40}{9} + \frac{6}{9} = 8$$

Remainders:

$$8 = (1 + 5 + 8 + 2 + 4 + 6) \div 9$$

For brevity let us call the remainders **"overs." The overs of a sum will equal the sum of the overs of the addends :** so the over of a product will equal the product of the overs of the factors, the 9s being thrown out in each case.

(*a*) Find the " 9 over " of 428357642.

For convenience we will space the figures thus :

4	2	8	3	5	7	6	4	2
	6	14						
		5	8	13				
				4	11			
					2	8	12	
							3	5

(See Explanation, p. 28.)

(*b*)

2, 4, 1, (6)	382965
1, 2, 6, (6)	453258
3, 6, 3, **3**	245769
(3=3) 3	1081992

When several addends are to be analyzed, do not set down each over, but add it to the digits of the next addend, until all the digits have been added. The last over should equal the over of the sum.

Explanation of (*a*).—4 and 2 are 6; 6 and 8 are 14. The digits of 14—viz., 1 and 4—when added, make 5. (This equals the over of 14 divided by 9.) Proceeding with 5, we may say: 5 and 3 are 8; 8 and 5 are 13. Now, adding the digits of 13, we have 4. Proceeding, we may say: 4 and 7 are 11. The digits of 11, taken together, make 2; etc., etc.

Explanation of (*b*).—The figures on the left followed by commas are obtained as follows: $3+8=11$. Now, 1 and 1 are the digits of 11; their sum is 2 (see the first explanatory figure at the left). Again, $2+2+9=13$; the digits of 13, *added*, equal 4 (see the second explanatory figure). Now, 4 and 6 are 10; the sum of the digits is 1 (see the third explanatory figure). Proceeding, we have $1+5$, or 6 (see the parenthetical figure). This 6 we add to the next addend below, and, taking the left-hand figure (4) in connection with it, we have 10, the sum of whose digits is 1 (see the left-hand figure of the second rank). The last 3 (printed in heavier type) is the final over. This equals the over of 1081992; therefore the work is supposed to be right. We say "supposed," for an error of 9 or its multiple would not be detected by this method.

PROOF OF MULTIPLICATION.

4675	The over of the multiplicand is 4
923	" " multiplier " 5
14025	" " 5×4 is 2
9350	
42075	
4315025	" " the product is 2

11025	The over is 0
1646	" " 8
18201150	" " $0=(8\times0)$

The proof of Division by casting out the nines depends on the fact that the dividend is always equal to the product of the divisor and quotient, plus the remainder.

1. Required to prove that the quotient of 683475 divided by 29 is 23568, the remainder being 3.

OPERATION.

29)683475(23568

Over, 2, ... Over, 6 ; $2 \times 6 = 12$; over, 3.

$683475 - 3 = 683472$; "over," 3.

RULE.—*Multiply the over of the divisor by the over of the quotient, and cast the 9s out of the product. Subtract the remainder from the dividend and cast the 9s out of the result. The over should equal the over last obtained.*

For further information, see PART FIRST of this treatise. This method of proving the four simple rules is of great practical importance, as it is easy, rapid, and reasonably certain. No pains should be spared to induce pupils to form a habit of proving their own work. Their future safety and success may depend upon this practice. No person can be considered a sound business man who does not habitually prove his work.

The Greatest Common Divisor.

TO FIND THE GREATEST COMMON DIVISOR (G. C. D.)

85. RULE.—*Divide the greater of the two given numbers by the less; then the divisor by the remainder; then divide the last divisor by the last remainder, and continue till there is no remainder. The last divisor will be the G. C. D.*

If there be more than two numbers whose G. C. D. is required, find the G. C. D. of either two of the numbers; then

the G. C. D. of this exact divisor and one more of the numbers; then the G. C. D. of this last measure, and still another of the numbers, until every number has been considered.

Generally, the very last division mentioned under the rule may be omitted, as the result is usually obvious: as in finding the G. C. D. of 602 and 1032.

In this example it is plain that the last mechanical division of 172 by 86 might have been omitted, as a glance would show that there would be no remainder.

```
602)1032(1
     602
     430)602(1
         430
         172)430(2
             344
              86)172(2
                 172
                 000
```

86. ***The Least Common Multiple*** (L. C. M.) of two or more numbers is the least number which contains each of the given numbers as a factor.

87. It comprises all the prime factors of each.

88. To find the Least Common Multiple.

Rule.—*From the list of numbers given, omit every one which is a divisor of either of the others. Write those remaining in a line of dividends. Divide by any prime number that is exactly contained in two or more of them, and set the quotients and undivided numbers in a line beneath. Omitting again all such as may be divisors of others in the line, proceed with the second line as with the first, and continue to divide each line until no prime number is exactly contained in more than one of the numbers. Finally, take the continued product of the divisors and undivided numbers for the L. C. M.*

Find the L. C. M. of 18, 2, 3, 4, 75, 16, 42, 36, 385, 48.

Divisors.					
5	385	75	48	42	36
3	77	15	48	42	36
7	77	5	16	14	12
2	11	5	16	2	12
2	11	5	8	1	6
	11	5	4	1	3

L. C. M.$=5\times3\times7\times2\times2\times11\times5\times4\times3=277200$.

89. To find the L. C. M. of *two* large numbers, divide either by their G. C. D., and multiply the quotient by the other number—*e. g.*, G. C. D. of 14758 and 9263 is 157; $14758\div157=94$; $94\times9263=870722$, or L. C. M.

90. The L. C. M. is important in the addition and subtraction of fractions.

Questions for General Review.

91. Define the term Unit, Number, Arithmetic, Notation, Numeration, Order, Decimal, Tens, Period, and Digit. Name the first eight periods. Name the 3d Order; the 4th; the 10th; the 14th; the 17th; the 2d; the 5th; the 9th; the 18th; the 7th; the 17th. Why must ciphers be written in certain cases? In what order do hundreds stand? Millions? Hundreds of thousands? Thousands? How many figures are required to write 41 millions? 213 billions? Define *Plus*. What is meant by Sum? Pronounce 7, and add by 6s to 61. Pronounce 3, and add by 4s to 39. Pronounce 11, and add by 8s to 107. Recite Axiom I. Define "Concrete Number." Recite Axiom II.; Axiom III. Define "Tenth"; "Fourth." Recite the Table of U. S. money. Are accounts kept in mills, dimes, and eagles? What is meant by a census? A city? A county? Define Subtraction, Subtrahend, Minuend, Remainder, *Minus*. Repeat Axiom IV. What is the abbreviation for Bushels? Pecks? Dimes? Dollars? Quarts? Pounds sterling? Shillings? Pence? Define

Multiplication; Factor; Prime Number; Composite Number; Product; Multiplicand. Show that a product of two numbers is the same if either be taken as a multiplier. Define Division; Dividend; Measure; Common Measure; G. C. D.; Multiple; Common Multiple; L. C. M. State this problem, viz.: "Multiply the difference between 78 and 49 by 3; from the product subtract the difference between 128 and 124; to the remainder add 43, and divide the sum by 6." Find the Greatest Common Divisor (G. C. D.) of 849 and 1132; also their L. C. M. How is Division proved? How Multiplication? How are the 9s cast out in proving Addition? How in proving Multiplication? Division? Repeat Axiom VI. What are the four definitions of Division? Repeat Axiom VII. Repeat the four *principles* of Division, and illustrate them.

Miscellaneous Questions.

1. A grocer expended $2800 for sugar, rice, and beans, paying $22 per bbl. for sugar, $12 for rice, and $6 for beans, purchasing an equal number of bbls. of each. How many bbls. did he purchase?

2. A tailor bought an equal number of yds. of three kinds of cloth; the price of the first was $2; of the second, $5; of the third, $7. If he purchased 18 yds. of the first kind, how much money did he expend in all?

3. A farmer exchanged 42 sheep, valued at $3 each, for wheat worth $2. How much did he receive?

4. If the sum of two numbers is 275, and one of the numbers is 97, what is the other?

5. If the greater of two numbers is 225, and the difference is 102, what is the smaller?

6. If the difference of two numbers is 106, and the smaller is 38, what is the larger?

7. If the sum of the ages of three men is 105 years, and the eldest is 5 years older than the second, and the second 5 years older than the youngest, what is the age of each?

Solution. $105-(5+5+5)$, or $90\div3$=age of youngest.

8. If a watch and chain together cost \$125, and the difference of their respective prices was \$65, what was the cost of each? *Solution.* $(125\div65)\div2$=cost of the chain.

9. The product of two numbers is 264; the product of one of the numbers multiplied by 4 equals 48; what is the other number?

10. Allowing 3 pickets to the foot, how many pickets will fence a lot 75 feet by 25 feet?

11. How many pickets will be required to fence a lot 40 feet by 90 feet, allowing 5 pickets for every two feet?

12. From the sum of 9983, 10002, and 8587, take 999×24, and divide the remainder by 6.

13. If 48 bushels of wheat are worth \$96, what are 81 bushels worth?

14. If a man buy 15 bushels of apples at 35 cents and 23 bushels at 38 cents, what will he gain if he sell the whole at 40 cents?

15. A merchant collected from A \$250, from B \$318, and from C as much as from A and B both, lacking \$25. If he were to expend the amount for flour at \$11 per bbl., how many barrels would he buy?

16. A manufacturer having 1080 yards of carpeting of one kind, 680 yards of another kind, 360 of another, and 480 of another, divided the several parcels into pieces, so that the lengths of all the pieces were equal. (G. C. D.)

17. If a landholder has one lot of 2112 acres, another of 1344 acres, another of 4800 acres, and a fourth lot of 17760 acres, and so divide each separate lot that all the farms laid out shall be equal, and as large as possible, how many farms will there be and what will be their size? (G. C. D.)

18. A person purchased an equal number of exact lbs. of almonds, raisins, figs, and candy. The prices were respectively 24 cts., 20 cts., 32 cts., and 42 cts. What was the least possible amount of cash necessary? (L. C. M.)

19. A carpenter desired to side up certain houses, which

were respectively 14, 28, and 42 feet long, with clapboards of such a length that none need be cut. Of what length were the boards to be purchased?

20. Judging from the first 16 inscriptions on the stones of a certain cemetery, on which the ages of the departed were stated in years as 5, 50, 16, 92, 83, 2, 21, 34, 17, 1, 85, 101, 49, 3, 68, 58, at what average age did the people of the village die?

21. The attendance of a certain class was as follows: Monday 32, Tuesday 36, Wednesday 34, Thursday 38, Friday 36. What was the average attendance?

22. What is the least capacity of a vessel which will contain an exact number of pints, half-gallons, and barrels?

23. What was the mean temperature (average of heat) when the thermometer marked 54° 68° 32°, 30° 49° 64°, 43° 58° 40°, during three days?

COMMON FRACTIONS.

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92. A fraction expresses one or more equal parts of a unit.

93. Any number less than one must be fractional. **When, in division, the divisor is not exactly contained,** the result is a fractional number.

94. Every fraction expresses a problem in Division, in which the dividend is called the **Numerator,** and the divisor is called the **Denominator,** and the quotient is called **the Value of the fraction (Art. 57).** The numerator shows **how many** parts are taken; the denominator **names the**

parts. The numerator and denominator are the **terms of the fraction.** A **simple fraction** has but two terms, both integers.

95. The Numerator, considered as a whole, is called the **Unit of the fraction.**

96. *One of the equal parts* into which the numerator is divided is called a **Fractional unit.** In the fraction $\frac{2}{3}$, 2 is the unit of the fraction, and $\frac{1}{3}$ is a fractional unit.

97. The nature of the **different kinds of fractions** may be illustrated as follows: **Proper** fractions, $\frac{1}{2}$, $\frac{4}{5}$; **Improper** fractions, $\frac{2}{2}$, $\frac{5}{4}$; **Compound** fractions, $\frac{2}{7}$ of $\frac{3}{9}$, $\frac{3}{5}$ of 4; **Complex** fractions, $\frac{2}{\frac{3}{5}}$, $\frac{\frac{3}{2}}{4}$, $\frac{\frac{3}{4}}{\frac{5}{8}}$; **Mixed** numbers, $4\frac{2}{7}$, $5\frac{1}{6}$.

98. When the parts have the same name, the fractional units are alike and the **fractions are similar.**

99. As the quotient obtained from dividing any number by 1 is always equal to the number so divided, **any integer may be represented by a fraction** whose numerator equals the integer and whose denominator is 1.

100. *General Principles of Fractions.*

From Art. **57** and Art. **62** we deduce the following:

To MULTIPLY A FRACTION,	*Multiply the numerator, or* *Divide the denominator.*
To DIVIDE A FRACTION,	*Divide the numerator, or* *Multiply the denominator.*
To REDUCE A FRACTION,	*Multiply both terms, or* *Divide both terms.*

101. The effect of thus operating on the terms of a fraction would also be obvious from the statement that the **numerator** indicates the *number* of parts, and the **denominator** the *size* of the parts.

Reduction of Fractions (Art. 33).

102. To Reduce an Improper Fraction to an Integral form.

Perform the division as indicated. Thus, $\frac{14}{5}=2\frac{4}{5}$.

103. To Reduce a Mixed Number to an Improper Fraction.

Multiply the integer by the denominator, and set the product over the denominator. In reducing a mixed number, the numerator must be added to the product. Thus, $2\frac{4}{5}=\frac{2\times5+4}{5}$, or, $\frac{14}{5}$.

104. To Reduce a Fraction to its Lowest Terms.

Divide both terms by their G. C. D.

Illus.: $\frac{296\div37}{407\div37}=\frac{8}{11}$. Here 37=G. C. D.

105. To Reduce an Integer to a Fractional form.

I. *Write the integer with* 1 *for its denominator.*
II. *Multiply both terms by the required denominator.*

Illus.: Reduce 7 to tenths: $\frac{7}{1}$. $\frac{7}{1}\times\frac{10}{10}=\frac{70}{10}$.

106. To Reduce a Compound Fraction to a Simple One.

Note.—The numerator of a compound fraction constitutes a dividend made up of detached factors, and its denominator forms a divisor made up of detached factors.

Rule.—*First reduce whole or mixed numbers, if there be any, to improper fractions, and then apply the Rule under Art.* 67.

Illus.: $\frac{9}{40}$ of $\frac{8}{27}$ of $\frac{3}{5}$ of $\frac{15}{21}=\frac{1}{5}$ of $\frac{1}{3}$ of $\frac{1}{1}$ of $\frac{3}{7}=\frac{1}{35}$.

107. TO REDUCE FRACTIONS TO A COMMON DENOMINATOR.

NOTE.—The least common denominator (L. C. D.) of several fractions equals the L. C. M. of their denominators. Compound fractions must be simplified.

RULE.—*Divide the L. C. D. by the denominator of each fraction, and multiply the several quotients by the respective numerators. Write the products over the L. C. D.*

Illus.: Reduce $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}$ to their L. C. D. The L. C. M. of the denominators is 24.

$\frac{24}{8}=3$; and $3\times3=\ 9$, or the numerator of the 1st fraction.
$\frac{24}{6}=4$; and $4\times5=20$, " " " 2d "
$\frac{24}{12}=2$; and $2\times7=14$, " " " 3d "
Hence $\frac{3}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{2}{3}$, $\frac{1}{2}$, $\frac{3}{4}=\frac{9}{24}$, $\frac{20}{24}$, $\frac{14}{24}$, $\frac{16}{24}$, $\frac{12}{24}$, $\frac{18}{24}$.

108. TO REDUCE A COMPLEX FRACTION TO A SIMPLE FRACTION.

Multiply the outside terms for a numerator, and the middle terms for a denominator. Thus:

$$\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{18}{20};\ \frac{4}{\frac{3}{5}}=\frac{\frac{4}{1}}{\frac{3}{5}}=\frac{20}{3};\ \frac{\frac{4}{7}}{9}=\frac{\frac{4}{7}}{\frac{9}{1}}=\frac{4}{63}.$$

(See Division of Fractions.)

PROBLEMS IN REDUCTION OF FRACTIONS.

Reduce the following improper fractions to integers or mixed numbers:

1–6. $\frac{21}{10}$; $\frac{42}{21}$; $\frac{18}{6}$; $\frac{29}{6}$; $\frac{342}{70}$; $\frac{248}{31}$;
7–12. $\frac{96}{8}$; $\frac{315}{15}$; $\frac{100}{27}$; $\frac{513}{133}$; $\frac{1261}{356}$; $\frac{48}{3}$;
13–17. $\frac{1001}{11}$; $\frac{18008}{13}$; $\frac{4004}{13}$; $\frac{9009}{7}$; $\frac{3003}{11}$;
18–22. $\frac{217683}{9}$; $\frac{5427}{9}$; $\frac{168}{3}$; $\frac{1548}{4}$; $\frac{10576}{8}$.

Reduce the following mixed numbers to improper fractions:

1–5. $22\frac{7}{16}$; $31\frac{9}{25}$; $44\frac{9}{59}$; $3\frac{2}{125}$; $27\frac{2}{13}$;
6–10. $1000\frac{9}{1000}$; $102\frac{4}{102}$; $7\frac{3}{7}$; $100\frac{7}{10}$; $6\frac{5}{19}$;

Reduce the following to their lowest terms:

1–5. $\frac{363}{605}$; $\frac{333}{396}$; $\frac{875}{1008}$; $\frac{80}{96}$; $\frac{924}{1188}$;

6–10. $\frac{814}{1221}$; $\frac{2253}{3004}$; $\frac{1599}{2665}$; $\frac{322}{1127}$; $\frac{591}{788}$.

Reduce the following integers to fractional forms:

1–5. 8 to thirds, tenths, fifteenths, fifths, sixths.

6–10. 124 to thirds, tenths, fifteenths, fifths, sixths.

Reduce the following compound fractions to simple ones:

1–4. $\frac{2}{3}$ of $\frac{3}{4}$; $\frac{5}{7}$ of $\frac{2}{20}$; $\frac{9}{26}$ of $\frac{13}{27}$; $\frac{3}{5}$ of $2\frac{1}{2}$;

5, 6. $\frac{6}{7}$ of $\frac{28}{36}$ of $5\frac{1}{9}$; $\frac{11}{51}$ of $\frac{117}{203}$ of $16\frac{9}{83}$.

Reduce the following to a common denominator; viz., make them *similar*:

1–4. $\frac{2}{5}$, $\frac{3}{8}$, $\frac{5}{16}$; $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$; $\frac{7}{11}$, $\frac{8}{12}$, $\frac{9}{13}$; $\frac{6}{27}$, $\frac{4}{7}$;

5–8. $\frac{5}{32}$, $\frac{7}{34}$, $\frac{9}{36}$; $\frac{2}{7}$, $\frac{8}{9}$, $\frac{10}{11}$; $\frac{1}{6}$, $\frac{2}{7}$, $\frac{3}{8}$; $\frac{11}{12}$, $\frac{12}{22}$.

Arrange in order of value or magnitude the following:

9–11. $\frac{2}{3}$, $\frac{4}{9}$, $\frac{3}{5}$, $\frac{7}{11}$; $\frac{5}{11}$, $\frac{9}{22}$, $\frac{15}{33}$; $\frac{9}{10}$, $\frac{4}{5}$, $\frac{24}{30}$, $\frac{7}{10}$.

Find the simplest expressions for the following:

1–4. $1\frac{2}{9}\div\frac{3}{5}$; $2\frac{4}{7}\div\frac{1}{3}$; $\frac{1}{2}$ of $\frac{3}{5}\div2\frac{1}{11}$; $\frac{6}{7}\div\frac{7}{6}$;

5–10. $\dfrac{4\frac{1}{5}}{1\frac{1}{2}}$; $\dfrac{6}{\frac{3}{4}}$; $\dfrac{\frac{3}{4}}{6}$; $\dfrac{8\frac{1}{9}}{2}$; $\dfrac{\frac{2}{3}}{11}$; $\dfrac{5\frac{1}{4}}{6}$.

Addition of Fractions.

109. To Add Fractions.

Make them similar, and then write the sum of the numerators over the common denominator. When one or more mixed numbers are given as addends, find the sum of the integers separately.

Illustration I.

$$10\tfrac{1}{5}+3\tfrac{3}{4}+\tfrac{5}{6}+7\tfrac{2}{15}+2=\,?$$

a. $22+\dfrac{12+45+50+8}{60}=$

b. $22+\frac{115}{60}=22+1\frac{55}{60}=$

c. Answer, $23\frac{11}{12}$.

Illustration II.

$4\frac{5}{7}+\frac{25}{12}$ of $1\frac{1}{5}+\frac{25}{4}$ of $2\frac{7}{10}$ of $\frac{1}{5}=$?

a. $4\frac{5}{7}+(\frac{25}{12}$ of $\frac{6}{5})+(\frac{25}{4}$ of $\frac{27}{10}$ of $\frac{1}{5})=$

b. $4\frac{5}{7}+\quad 2\frac{1}{2}\quad +\quad 1\frac{7}{8}\quad =$

c. $7\ +(\frac{5}{7}+\frac{1}{2}+\frac{7}{8})=7+\frac{40+28+49}{56}=$

d. $7\ +\frac{117}{56}$. *e.* $7+2\frac{5}{56}=$

Answer, $9\frac{5}{56}$.

110. *a.* Mark off each complete fraction. *b.* Simplify such as are compound. *c.* Reduce the simple fractions to their L. C. D. *d.* Add similar fractions, and reduce the sum. *e.* Annex the proper fraction to the sum of the integers.

Subtraction of Fractions.

111. TO FIND THE DIFFERENCE BETWEEN TWO FRACTIONS.

Make the fractions similar. Write the difference between the numerators over the C. D.

NOTE.—In the case of *mixed numbers* (*a*), if the fraction in the subtrahend is less than that of the minuend, *annex the difference between the fractions to the difference between the integers.* (*b*) If the fraction of the subtrahend be the larger, add a unit to the fraction of the minuend, and, after finding the remainder, annex it to the difference between the integers less one. *Illus.:* (*a*) $4\frac{3}{7}-2\frac{2}{7}=2\frac{1}{7}$; (*b*) $4\frac{2}{9}-2\frac{2}{7}=(3-2)+(1\frac{2}{9}-\frac{2}{7})=1+(\frac{77}{63}-\frac{18}{63})=1\frac{59}{63}$. Or $(3-2)+\frac{11}{9}-\frac{2}{7}=1+\frac{59}{63}$.

Multiplication of Fractions.

112. WHEN ONE FACTOR IS A MIXED NUMBER.

First multiply the fraction, then multiply the integer, and, lastly, add the two products.

Illus.: $4\frac{1}{5}\times 7=28+\frac{7}{5}$; $9\times 2\frac{3}{5}=18+\frac{27}{5}$; or $23\frac{3}{5}$.

113. When both Factors are Fractions.

First cancel, if possible; then take the product of the numerators for a numerator, and the product of the denominators for a denominator. Mixed numbers should be reduced to improper fractions.

Exercise VIII.

Find the value of each of the following:

1–6. $\frac{3}{7}\times\frac{2}{5}$; $\frac{4}{5}\times\frac{3}{11}$; $\frac{1}{2}\times\frac{2}{1}$; $\frac{1}{2}\times\frac{1}{2}$; $\frac{2}{3}\times\frac{1}{4}$; $\frac{2}{3}\times\frac{3}{4}$.

7–11. $2\frac{1}{5}\times\frac{4}{7}$; $3\frac{1}{7}\times12\frac{4}{9}$; $4\frac{11}{13}\times\frac{2}{13}$; $3\frac{1}{11}\times\frac{12}{17}$; $8\frac{1}{4}\times3\frac{10}{15}$.

12–16. $\frac{2}{7}\times1\frac{1}{2}$; $\frac{3}{8}\times2\frac{11}{15}$; $\frac{5}{11}\times\frac{4}{15}$; $\frac{4}{9}\times3\frac{1}{4}$; $19\frac{1}{12}\times16\frac{1}{9}$.

Division of Fractions.

114. Invert the divisor, and proceed as in multiplication of fractions. (See *General Principles.*)

The division of one fraction by another is best explained by first reducing the two fractions to a common denominator. *Fractions that have a common denominator are to each other as their numerators.*

Illus.: $\frac{4}{7}\div\frac{2}{3}$. $\frac{4}{7}=\frac{12}{21}$; $\frac{2}{3}=\frac{14}{21}$, $\frac{12}{21}\div\frac{14}{21}=12\div14=\frac{12}{14}$, or $\frac{6}{7}$; $\therefore$ $\frac{4}{7}\div\frac{2}{3}=\frac{6}{7}$.

Exercise IX.

1–16. Divide the first fraction by the second in each example of Exercise VIII.

17–22. $\dfrac{\frac{8}{12}}{\frac{4}{36}}$; $\dfrac{5\frac{1}{2}}{\frac{13}{26}}$; $\dfrac{\frac{7}{15}}{\frac{7}{8}}$; $\dfrac{1}{1\frac{1}{9}}$; $\dfrac{1}{\frac{3}{5}\text{ of }\frac{2}{3}}$; $\dfrac{\frac{9}{18}}{\frac{110}{126}}$.

Note.—In questions like 17–22, take the product of the outside terms for a numerator, and the product of the middle terms for a denominator. Thus:

17. $\dfrac{\frac{8}{12}}{\frac{4}{36}}=\frac{288}{48}$; by cancellation, $\dfrac{\frac{2}{3}}{\frac{1}{9}}=\frac{18}{3}=6$. 20. $\dfrac{1}{1\frac{1}{9}}=\left(\dfrac{\frac{1}{1}}{\frac{10}{9}}\right)=\frac{9}{10}$.

115. To find what Fraction one Number is of Another.

Divide that which is taken as a part *by that of which it is a part. Whatever form the question may take, the di-*

visor is preceded by the word "of." The divisor is, of course, the measure or standard.

Illus.: What part of $\frac{4}{5}$ of a gallon is $\frac{7}{9}$ of a pint? $\frac{4}{5}$ gal. $=$ $\frac{32}{5}$ pts.; $\frac{7}{9} \div \frac{32}{5} = \frac{35}{288}$.

Exercise X.

1–5. What part of $\frac{4}{7}$ is $\frac{1}{9}$? Is $\frac{3}{11}$? $\frac{2}{3}$? $3\frac{1}{2}$? $\frac{1}{3}$ of $\frac{5}{7}$?

6–10. What part of $\frac{3}{14}$ is $\frac{2}{7}$? Is $4\frac{1}{2}$? $\frac{1}{2}$ of $\frac{4}{9}$? $\frac{112}{220}$? $\frac{349}{117}$?

THE STATEMENT OF PROBLEMS.

116. Before proceeding to solve a complex problem, set forth each and every process by means of fitting signs. Use parentheses for groups of factors or for compound fractions. Simplify every expression, especially if it is to be increased or diminished by another.

Write each simplified number, whether sum, difference, product, or quotient, under its equivalent expression.

Proceed with the second rank from left to right, as before. Finally, collect values.

Illus.: Add $\frac{30}{5}$ to $\frac{42}{7}$ of 3×14, and then divide the result by $\frac{40}{5}$.

a. $\dfrac{(\frac{42}{7} \times 3 \times 14) + \frac{30}{5}}{\frac{40}{5}} =$

b. $(6 \times 3 \times 14 + 6) \div 8 =$

c. $258 \div 8 =$

d. Ans. $32\frac{1}{4}$.

Exercise XI.

The pace, or double step, in walking measured 5 Roman feet; as, 1000 paces.

A Roman foot was $11\frac{62}{100}$ inches: 5 such feet was called a

pace, or double step. How many yards in a mile of 1000 Roman paces?

If 3 lbs. of tea are worth as much as 5 lbs. of coffee, 2 lbs. of coffee as much as 11 lbs. of sugar, and 3 lbs. of sugar are valued at 33 cents, what cost 1 lb. of tea?

Solution. As 1 lb. of tea = val. of $\frac{5}{3}$ lbs. coffee; and 1 lb. coffee = val. of $\frac{11}{2}$ lbs. of sugar; and as 1 lb. sugar = $\frac{33}{3}$ cents, then 1 lb. of tea = $\frac{5}{3}$ of $\frac{11}{2}$ of $\frac{33}{3}$ cents, or $100\frac{5}{6}$ cents.

If a man whose steps are equal, each being 30 inches, take 2 steps in a second, how much quicker can he walk a mile than if his steps were 25 inches long?

$$\textit{Solution.}\quad \frac{5280\times 12}{2\times(30-25)} = \text{No. of sec. gained.}$$

By the terms of a will, the eldest son was to receive one-third of an estate; each of the two younger sons one-seventh; each of three daughters, one-tenth; the remainder, $680, was devised to a hospital. Required the total.

Find the difference between the product and quotient of $2\frac{1}{4}$ and $1\frac{1}{8}$ and divide it by three and one-sixth times $\frac{4}{7}$ of 20.

Simplify $\frac{2\frac{1}{3}}{7} + \frac{4}{10\frac{1}{2}} - \frac{8}{15}$ of $\frac{3}{28}$ and divide the result by $4\frac{6}{7}$.

Divide the product of $3\frac{1}{2}$ and $7\frac{2}{5}$ by their sum, and to the result add their difference.

To the first of three persons John gave $\frac{1}{4}$ of his money; to the second, he gave $\frac{2}{3}$ of the remainder; to the last he gave 9 cents; he still had 1 cent. What had he at first?

If A have $\frac{1}{2}$ of $2\frac{2}{3}$ as many cents as B, and C have $2\frac{1}{11}$ as many as B, and B have $3\times\frac{3}{7}$ of $\frac{21}{9}$ of 99 cents, how many has each? What fraction of C's does A's equal?

DECIMAL FRACTIONS.

117. The **denominator of a decimal fraction** is understood to be 10, or some power of 10; as, 100, 1000, 100000.

118. The **numerator only** of a decimal fraction **is written. It must occupy as many places**—*i. e.*, consist of as many figures—as there are ciphers in the denominator. Thus, to express eight ten-thousandths, we write .0008, prefixing a decimal point and three ciphers to 8.

119. The **decimal point** is used to mark the fractional character of the expression, and, in mixed numbers, to separate the integral from the fractional parts; thus, $7\frac{4}{10}=7.4$.

120. To Write a Decimal Fraction.

Write the numerator so that the right-hand figure shall have the name of the denominator.

121. NOTE.—Pupils should consider, first, how many places are required; secondly, how many places the *given* numerator requires as an integer; and, thirdly, that the difference between these numbers indicates how many ciphers are to be prefixed. *Illus.:* Req. to write 42 ten-thousandths. I. Ten-thousandths require 4 places. II. 42 occupies 2 places. III. The difference between 4 and 2 indicates the need of 2 ciphers as prefixes. *Ans.* .0042.

122. To Read a Decimal Fraction.

Read the numerator, and name its right-hand order.

General Principles of Decimals.

123. I. Decimals arise from dividing ONE into ten equal parts, and from dividing each one of these *tenths* into ten equal parts, etc.

124. II. Decimals are written at the right of a separatrix or point, the numerator only being expressed.

125. III. The orders of decimals are numbered from the point, and the names correspond with those of whole numbers,

except that one figure less is required for the expression of each name.

126. IV. Every cipher inserted between the separatrix and the digits of a decimal diminishes their value tenfold.

127. V. Ciphers annexed to a decimal do not alter its value, as they do not alter the position of the significant figures.

128. To convert a decimal into a common fraction, *erase the point and write the denominator.*

129. Any whole number may be made to take the form of a mixed number, and any decimal may be made to assume a larger denominator by the suffixing of ciphers.

Table.

Hundred Thousands.	Ten Thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.
3	4	6	5	2	3	.	4	2	3	4	5	6

$$23.4561 = 2 \times 10 + 3 + \tfrac{4}{10} + \tfrac{5}{100} + \tfrac{6}{1000} + \tfrac{1}{10000} = 23\tfrac{4561}{10000}$$

$$5.123456 = 5 + \tfrac{1}{10} + \tfrac{2}{100} + \tfrac{3}{1000} + \tfrac{4}{10000} + \tfrac{5}{100000} + \tfrac{6}{1000000} = 5\tfrac{123456}{1000000}.$$

Exercise XII.

Write out the separate value of each figure of the following numbers: (1) 3.1416; (2) 7854; (3) .5236; (4) .433; (5) 1.72; (6) 2.598; (7) 3.6339; (8) 4.8284; (9) 11.196; (10) 39.374; (11) 2.2046; (12) 15.432; (13) 61.027; (14) 119.6046; (15) 35.3155; (16) .0338; (17) 1.4142;

(18) 2218.2; (19) 2150.4; (20) 277.27. (21–40) Write each number as a common fraction.

NOTE.—The above are important factors in Mensuration as well as in Denominate Numbers. Reference will be had to these again. See also the problems in

EXERCISE XIII.

Write in Arabic figures as decimals:

1–8. Eleven thousand seven hundred and eighty-five hundred-thousandths; four thousand seven hundred and fourteen ten-thousandths; seven and decimal sixty-six thousand three hundred and twelve hundred-thousandths; two and decimal eighteen hundredths; four and decimal four thousand eight hundred and sixty-six ten-thousandths; twenty-three and decimal eighty-five hundredths; one hundred and ninety-three thousandths; twenty-four, and thirty-three hundredths.

9–20. Fifteen hundredths; six millionths; four hundred and twelve hundred-millionths; ten, and seventeen hundred-thousandths; one thousand and thirteen, and one hundred and sixty-one ten-millionths; two, and twenty-nine hundredths; forty-eight thousandths; two tenths; nine, and four thousand one hundred and fifty-eight ten-thousandths; three millionths; three thousandths; three, and three billionths.

EXERCISE XIV.

Express the following as common fractions:

1–15. .024; 3.15; .2; 14.17; .016; .00016; 1.0158; 19.009045; 4568.35; .456835; .0018; 56.56; .5656; .05656; 5.656.

EXERCISE XV.

Write out the following in decimal form:

1–18. $32\frac{154}{1000}$; $\frac{2}{10}$; $\frac{416}{10000}$; $\frac{71}{1000}$; $\frac{3}{1000000}$; $\frac{31416}{100000}$; $\frac{59}{10000}$; $\frac{841}{1000}$; $\frac{5}{100000000}$; $11\frac{4298}{100000}$; $91\frac{56}{1000}$; $3\frac{4}{10}$; $5\frac{4}{1000}$; $8\frac{9}{100}$; $1641\frac{1641}{1000000}$; $705\frac{705}{1000}$; $39\frac{39}{10000}$; $18\frac{18}{100000}$.

130. Conversion of Common Fractions into Decimals.

If both terms of a fraction be multiplied by the same number, the result is equal to the original fraction.

Multiply the terms by any number that will change the denominator to that of a decimal fraction, and then write the fraction decimally, thus :

Illus. a. $$\frac{1\times125}{8\times125}=\frac{125}{1000}=.125$$

Illus. b. $\frac{3}{5}=.6$; for $5)3\times10(6$, and $6\div10=.6$.

$\frac{3}{16}=.1875$; for $16)3\times10000(1875$, and $1875\div10000=.1875$.

By annexing ciphers to the numerator, the dividend is multiplied. After the first division has been performed, the quotient must be divided in order that the result may be correct. The division is effected by pointing off one or more figures at the right hand of the quotient as a decimal.

From illustrations *a* and *b* we deduce the following :

To Convert a Common Fraction into a Decimal.

Rule.—*Annex ciphers to the numerator, divide by the denominator, and point off in the result (counting from the right) as many figures as there are ciphers annexed.*

Note.—When the denominator is not exactly contained, do not in general protract the division beyond the fourth or fifth cipher annexed.

Exercise XVI.

(1) $\frac{3}{12}$; (2) $\frac{3}{8}$; (3) $\frac{5}{16}$; (4) $\frac{2}{80}$; (5) $\frac{9}{160}$; (6) $2\frac{18}{144}$; (7) $16\frac{1}{256}$; (8) $\frac{180}{576}$; (9) $\frac{1}{2}$ of $\frac{5}{9}$ of $\frac{3}{4}$; (10) $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{6}}{\frac{2}{5}+\frac{3}{8}+\frac{1}{3}}$; (11) $\dfrac{\frac{5}{6}\times\frac{2}{7}}{5}$; (12) $\dfrac{3\frac{1}{4}}{5}$; (13) $\dfrac{6}{5\frac{1}{7}}$; (14) $\dfrac{\frac{2}{7}\times\frac{3}{4}}{1\frac{1}{5}+3\frac{2}{9}}$; (15) $\dfrac{16\frac{1}{5}-3\frac{1}{10}}{8\frac{1}{2}}$; (16) $\dfrac{9\frac{1}{5}+2\frac{1}{4}}{\frac{1}{2}\text{ of }\frac{2}{3}+\frac{10}{12}}$; (17) $\dfrac{\frac{26}{80}+4\frac{1}{7}}{(\frac{2}{5}+\frac{7}{8})\times4}$; (18) $\dfrac{\frac{5}{6}}{\frac{6}{5}}\times\dfrac{\frac{7}{8}}{\frac{3}{10}}$. (Art. 114.)

131. *Repetends.*

I. When the same figure or set of figures recur constantly in reducing a common fraction to a decimal, the result is called a repeating decimal, or, in brief, a Repetend. If the same figures recur from the beginning, they form a pure repetend. Thus: .333 . . . and 1515 . . . are pure repetends.

II. A repeating figure or single group of repeating figures is called a period. But one period need be written if a dot be placed over its first figure and another over its last. Thus: $.\dot{3}=333\ldots$, and $.\dot{1}\dot{5}=1515\ldots$

Such expressions as $.1\dot{3}$ and $.2\dot{1}\dot{5}$ are called mixed repetends. In these the first or non-repeating figures are common decimals.

III. If, after a common fraction has been reduced to its lowest terms, its denominator contains any other factor than 2 or 5, the result, when reduced, will be a repetend.

Exercise XVII.

Reduce the following to recurring decimals:

1–15. $\frac{6}{7}$; $\frac{11}{12}$; $\frac{7}{9}$; $\frac{5}{7}$; $\frac{7}{11}$; $\frac{5}{13}$; $\frac{4}{19}$; $\frac{2}{17}$; $\frac{4}{27}$; $\frac{10}{55}$; $\frac{14}{33}$; $\frac{1}{3}$; $\frac{5}{6}$; $\frac{7}{30}$; $\frac{15}{23}$.

As $\frac{1}{9}=.\dot{1}$, so $\frac{2}{9}$ will $=.\dot{2}$, and $\frac{7}{9}=.\dot{7}$; and as $\frac{1}{99}=.\dot{0}\dot{1}$, and $\frac{2}{99}=.\dot{0}\dot{2}$, and $\frac{55}{99}=.\dot{5}\dot{5}$; and as $\frac{1}{999}=.\dot{0}0\dot{1}$, and $\frac{26}{999}=.\dot{0}2\dot{6}$, and $\frac{348}{999}=.\dot{3}4\dot{8}$; and as $\frac{1}{9999}=.\dot{0}00\dot{1}$, and $\frac{64}{9999}=.\dot{0}06\dot{4}$, and $\frac{9426}{9999}=.\dot{9}42\dot{6}$, we may infer the following

Rule.—*For the true denominator of a pure repetend write as many 9s as there are recurring figures.*

The members of the following pairs are equals:

$.\dot{4}=\frac{4}{9}$; $.\dot{1}\dot{6}=\frac{16}{99}$; $.\dot{0}45\dot{1}=\frac{451}{9999}$; $.\dot{1}5\dot{6}=\frac{156}{999}$; $.\dot{0}0\dot{4}=\frac{4}{999}$.

IV. As the value of a decimal figure depends in part on its location, it is obvious that if $.\dot{6}=\frac{6}{9}$, $.0\dot{6}=\frac{6}{90}$, and $.00\dot{1}\dot{6}=\frac{16}{9900}$. The denominator of the repetend when decimal figures precede is as many 9s as there are recurring figures, followed

by as many ciphers as there are common decimal figures before the repetend.

V. In such an expression as $.1\dot{6}$ the first digit is a decimal figure and represents $\frac{1}{10}$, the second digit is a repetend, and as it stands in the second place it represents $\frac{6}{90}$. Now $(\frac{1}{10}+\frac{6}{90})=(\frac{9}{90}+\frac{6}{90})=\frac{15}{90}$. If from 16 we take the first digit, the remainder, 15, will be the numerator of the resulting common fraction whose denominator is the denominator of the repetend.

VI. To find what common fraction equals a given Mixed Repetend.

Rule.—*From the whole quantity subtract the part whose digits do not recur, and write the remainder over the denominator of the repetend.*

Illus. *a.* $.5\dot{4}\dot{4}=?$ $544-5=539$; $\frac{539}{990}=.5\dot{4}\dot{4}$.

Illus. *b.* $.15\dot{3}=?$ $153-15=138$; $\frac{138}{900}=.15\dot{3}$.

Illus. *c.* $27.10\dot{5}\dot{6}=?$ $1056-10=1046$; $27\frac{1046}{9900}=27.10\dot{5}\dot{6}$.

Exercise XVIII.

Find the exact values of the following:

1–12. $.0\dot{7}-.\dot{0}\dot{7}$; $.1\dot{6}-.\dot{1}\dot{6}$; $5\times\dot{2}5\dot{5}$; $5\times.2\dot{5}\dot{5}$; $4.1\dot{6}\div3$; $\frac{1}{7}+\frac{1}{11}$; $\frac{1}{9}+\frac{1}{17}$; $.\dot{2}+.\dot{4}\dot{6}$; $.\dot{8}-.\dot{3}1\dot{6}$; $.\dot{8}\dot{4}-.08\dot{4}$; $.1\dot{4}\dot{6}$; $.03\dot{2}$.

132. To Add Decimals.

Write the numbers so that the tenths shall form a column. Add similar orders as in Addition of Integers. Point off as many places in the amount as equal the largest number of places found in any of the addends. (Probs., Art. 137.)

133. Dimes, cents, and mills, in U. S. currency, are expressed decimally. Dollars are integers, dimes are tenths, cents are hundredths, and mills are thousandths. Thus, 12 dollars, 14 cents, and 7 mills are written $12.147.

134. To Subtract one Decimal Number from Another.

Rule.— *Write the number having the smaller integral part under the other, so that tenths may stand under tenths, hun-*

dredths under hundredths, etc. Subtract as in whole numbers, and point off as in addition of decimals. (Art. 137.)

135. TO MULTIPLY WHEN EITHER OR BOTH FACTORS ARE DECIMALS.

Multiply as in whole numbers, and point off in the product as many decimal figures as there are in both factors, prefixing ciphers if necessary. (Problems, Art. 137.)

For method of contracting, see Part First, Art. **249.**

136. GENERAL RULE FOR DIVISION OF DECIMALS.

Make at least as many places in the dividend as there are in the divisor, by annexing ciphers.

Divide as in whole numbers.

If there be a remainder after bringing down all the figures of the dividend, annex ciphers to it and continue the division as far as may be thought proper. The ciphers thus annexed represent additional decimal places in the dividend.

Finally, point off as many decimal places in the quotient as, counted with those in the divisor, will equal the number of decimal figures in the dividend.

137. *Table of Decimals for Miscellaneous Problems.*

a	.06	*f*	.216	*k*	5.236	*p*	.528
	.02		.482	*l*	1.728		.484
	.735	*g*	.112		1.19	*q*	.3333
b	.801		.299		277.274		7.44
	.017	*h*	.608	*m*	.231	*r*	735.812365
c	56.345		2.006		.282	*s*	25.72046
	42.3		.032		2.6	*t*	61.0008719
	7.615	*i*	4.545	*n*	61.028	*u*	7.7888
d	114.568		7.854		5.76	*v*	4.4444
e	12.37	*j*	31.416	*o*	4.865	*w*	6.789023
	7.615		.2134		486.5		678.9023
	.009		21.34		48.65		678902.3
	23.153		2.134		.04865		6789.023

Exercise XIX.

Examples illustrative of the use of the *Table:*

1. Add *r*, *s*, *t*, *u*, *v*, and *w*.
2. Add *k* and the six following numbers.
3. Add *a* and the nine following numbers.
4. Add all that follow *o*.
5. Add all that precede *f*.
6. Add *e* and the next four.
7. Add *p* and the next five.
8. Add the first column.
9. Add the third column.
10. Add the last column.

1. From *a* subtract the next number.
2. From *b* subtract the next below.
3. From *c* subtract the next.
4. From *d*, the next.
5. From *j*, the next.
6. From *p*, the next.
7. From *r*, the next.
8. From *u*, the next.
9. From *w* take *v*.
10. From *s* take *u*.

1. Multiply *a* by *b;* 2. *c* by *d;* 3. *e* by *f;* 4. *g* by *h;* 5. *i* by *j;* 6. *k* by *l;* 7. *m* by *n;* 8. *o* by *p;* 9. *q* by *r;* 10. *s* by *t*.

1. Divide *c* by *b;* 2. *d* by *h;* 3. *e* by *i;* 4. *q* by *a;* 5. *j* by *m;* 6. *n* by 2.2; 7. *m* by *l;* 8. *p* by *q;* 9. *j* by 4; 10. *l* by 12. (*a* includes .06, .02, .735; *b*, .801, .017, etc.)

138. *United States Currency.*

The sign $, prefixed to a number, indicates U. S. currency. Dollars, according to law, are units. Dimes, cents, and mills being decimals, are therefore to be written as tenths, hundredths, and thousandths.

In the result or answer five or more mills are considered as a cent, and less than five are ignored. Accounts are usually kept in dollars and cents. (See *Bills.*)

DENOMINATE NUMBERS (D. N.)

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139. A D. N. is a concrete number expressive of one or more units of length, surface, volume, weight, time, or value. A **Simple** D. N. shows but one kind of unit. A **Compound** D. N. expresses several denominations of a similar nature.

140. A **Table** shows the comparative values of similar numbers.

141. A **Standard Unit** is that of which the numbers in a table are fractions or multiples. Thus, in U. S. currency the dollar is the standard unit; in cloth measure, it is the yard; in land measure, the acre.

142. **Abbreviations** are used for the names of coins, weights, etc., and generally consist of the initials of the names in question.

The fundamental table is that from which all others are derived. It is the table of linear measure.

143. *Table of U. S. Money.*

Eagle, e.		Dollars, $.		Dimes, d.		Cents, c.		Mills, m.
1	=	10	=	100	=	1000	=	10000

144. *Table of English Money.*

		Pound, £.		Shillings, s.		Pence, d.		Farthings, qr.
$4.8665	=	1	=	20	=	240	=	960

145. *Table of French Money.*

		Franc, fr.		Decimes, d.		Centimes, cent.
$.193	=	1	=	10	=	100

146. *a, b, c.* *Tables of English Weights.*

a. TROY.—(1 lb. = 5760 gr.)

Pound, lb.		Ounces, oz.		Pennyweights, dwt.		Grains, gr.
1	=	12	=	240	=	5760

b. AVOIRDUPOIS.—(1 lb. = 7000 gr.)

Ton, t.		**Hundredweights, cwt.**		**Quarters, qr.**		**lbs.**		**oz.**		**Drams.**
1	=	20	=	80	=	2000	=	32000	=	512000

c. APOTHECARIES'.

Pound, lb.		Ounces, ℥.		Drams, ʒ.		Scruples, ℈.		Grains, gr.
1	=	12	=	96	=	288	=	5760

147. *Table of French Weights.*

	No. Grams.	Volume of Water.	Avoirdupois lbs.
Millier, or *stere*,	1000000	1 cubic metre, .	2204.6 pounds.
Quintal, . . .	100000	1 hectolitre, . .	220.46 "
Myriagram, . .	10000	10 litres, . . .	22.046 "
Kilogram, kilo,	1000	1 *litre* (1 quart),	2.2046 "
Hectogram, . .	100	1 decilitre, . .	3.5274 oz.
Decagram, . .	10	10 cu. centimetres	.3527 oz.
Gram, . . .	1	1 cu. centimetre .	15.432 grains.
Decigram, . .	$\frac{1}{10}$	$\frac{1}{10}$ do. . .	1.5432 grains.

SUMMARY OF THE METRIC SYSTEM.

Grades:	4	3	2	1	0	1	2	3
Values:	10000	1000	100	10	1	.1	.01	.001
Prefixes:	Myria.	kilo.	hecto.	deca.	**Unit.**	deci.	centi.	milli.
m. = 39.37 in.	.	.	.	.	**Metre.**			
l. = 61.03 cu. in.	.	.	.	.	**Litre.**			
kg. = $2\frac{1}{5}$ lbs.	.	.	.	.	**Gram.**			
a. = 119.6 sq. yds.		Hectare,			**Are,**		centiare.	
fr. = 19.36 cts.	.	.	.	.	**Franc.**			

148. *a*, *b*, *c*. *Tables of Lengths.*

a. LINEAR.—(Foot = Unit.)

Mile, mi.	Furlongs, fur.	Rods, rd.	Yards, yd.	Feet, ft.	Inches, in.
1 =	8 =	320 =	1760 =	5280 =	63360

b. SURVEYORS'.—(7.92 in. = 1 l.)

Mile, m.	**Chains,** ch.	Rods, rd.	Links, l.	Inches, in.
1 =	80 =	320 =	8000 =	63360

c. METRIC.—(m. = 3.2809 ft.)

Kilometre, km.	Hecto., hm.	Deca., dm.	Metre, m.	Deci., dm.	Cent., cm.	Milli, mm.
1 =	10 =	100 =	1000 =	10000 =	100000 =	1000000

149. *a*, *b*, *c*. *Tables of Surface.*

a. SQUARE.

Sq. mile,	Acre,	Roods,	Sq. rods,	**Sq. yds.,**	Sq. ft.,	Sq. in.
1 =	640 =	2560 =	102400 =	3097600		144 = 1 sq. ft.
	1 =	4 =	160 =	$160\times30\frac{1}{4}$ =	$160\times30\frac{1}{4}\times9$.	

b. LAND.

Sq. m.	**Acres,** a.	Sq. chains.	Poles, p.	Sq. links, sq. l.
1 =	640 =	6400 =	102400 =	64000000
	1	10	160	160×625.

c. FRENCH LAND.—(Unit or Are = 119.6 sq. yds.)

Hectare.	Are.	Centiares.
1 =	100 =	10000

150. *Tables of Capacity or Solidity.* (By SCALES.)

a. CUBIC, ENGLISH.

1728 cu. in. = 1 cu. ft.; 27 cu. ft. = 1 cu. yd.; 128 cu. ft. = 1 cord.

b. CUBIC, FRENCH (From Fr. Lin. meas.)

1000 cu. mm. = 1 cu. cent.; 1000 cu. cent. = 1 **cu. dec.**; etc.
$cm.^3 = .061$ cu. in.; **$d.^3$** $= 61.022$ cu. in.

c. French Liquid. (See Summary of the Metric System.)

A Litre = 1 cu. decim. =.908 dry qts. or 1.0567 liq. qts.

d. U. S. Liquid.

4 gills=1 pt.; 2 pts.=1 qt.; 4 qts.=1 **gal.**; 31½ g.=1 bbl.

The Unit = 1 gal. = 231 cu. in. distilled water (bar. 30° ther. 40°).

e. Dry Measure.

2 pts. = 1 qt.; 8 qts. = 1 pk.; 4 pks. = **1 bushel.**

U. S. bu. = 2150.42 cu. in.; Eng. Imp. bu. = 2218.19 cu. in.

151. *Table of Time.*

60 sec. = 1 min.; 60 min. = 1 hour; 24 hrs. = 1 day; 365¼ ds. = 1 year.

152. *Table of Circular Measurements.*

60 sec. (″)=1 min. (′); 60′ = 1 degr. (°); 360° = 1 circumfer. (cir.)

(For other facts see Remarks on the Tables, Appendix.)

It has long been the ardent wish of scholars and commercial men that the metric system should be adopted by all civilized nations. That desire seems about to be realized. We have, therefore, given prominence to the French system of weights and measures.

153. *Table of the New German Currency.*

As our monetary transactions with Germany are very extensive, we append a table of the new German currency. Its denominations are the Pfennig (pf.), the Groschen (gr.), and the Reichsmark, or "Mark."

10 Pfennige=1 Groschen; 10 Gr.=1 **Reichsmark**=$.238.

(Old currency, 1 Thaler=30 Groschen=360 pf. Legal value, 1871, of Prus. Th., 69 cts.; nom. value, 74.6 cts.)

The coinage consists of *copper* 1 and 2 pf.; *nickel* 5 and 10 pf.; *silver* 20 and 50 pf. (or 2 and 5 gr.); and also coins of 1 and 2 marks; and *gold* 10 and 20 Reichsmark pieces.

DENOMINATE NUMBERS.

154. Different names may be used to express the **same value.** Thus : 1 pound may be regarded as 16 ounces, or as 256 drams; $2\frac{1}{2}$ gallons, as 10 quarts, or as 20 pints; and 48 hours may be expressed as 2 days.

A **Simple D. N.** expresses units of one name only; a **Compound D. N.** consists of two or more parts having different names but similar in nature. Thus : 2 days 3 hours is a Compound D. N. 2 days 3 hours and 1 ounce is not a Compound D. N., but consists of *two* numbers. Quantities are usually expressed in **large denominations,** just as fractions are stated in the lowest terms.

Reduction of Denominate Numbers.

155. Reduction consists in resolving units of one name into units of another name.

156. The resolving of large units into smaller units is called **Reduction Descending.**

If it were required to find what a barrel of vinegar would sell for at 6 cents a pint, the number of pints in a barrel must first be obtained. Thus, 1 bbl.$=31\frac{1}{2}$ gal., or 126 qts., or 252 pints. These at 6 cents would sell for 1512 cents, or \$15.12.

Required the number of oz. in

2 t.	2 cwt.	2 qr.	2 lb.	2 oz.
	20	4	25	16
2	2	2	2	2

20	
42	cwt. (2 being added).
4	
170	qrs. " " "
25	
4252	lbs. " " "
16	
68034	oz. " " "

For convenience, write over each part of the compound number the number of its name required to make 1 of the next higher. In the example, the number of cwt. in the 2 t. is increased by the 2 extra cwt. given; so the number of qrs. is increased by the 2 extra qrs. The final result comprises all the parts of the given number.

To Reduce a Number of a Higher Order to one of a Lower Name.

Rule.—*Multiply the number of the highest denomination by the number of units of the next lower name contained in a unit of this highest; to the product add such as there are of the second denomination. Reduce this result in the same manner to the next lower denomination, and continue till the required denomination is obtained.*

Problems. **1-96.** Let **1, 2, 3, 4, 5,** and **6** each represent a D. N. of four parts having the names indicated by the respective abbreviations, and reduce each quartet to the lowest of its denominations. Thus, 1 *a* refers to 2 e. $2 1 ct. 1 mill.

1.	2	2	1	1	**2.**	3	3	1	1
3.	15	3	1	1	**4.**	19	4	1	1
5.	208	5	1	1	**6.**	1961	8	1	1

(*a*) e. $. c. m.; (*b*) £. s. d. qr.; (*c*) lb. oz. dwt. gr.; (*d*) qr. lb. oz. dr.; (*e*) ℥. ʒ. ℈. gr.; (*f*) kg. decg. g. dg.; (*g*) rd. yd. ft. in.; (*h*) ch. rd. l. in.; (*i*) m. dm. cm. mm.; (*j*) s.rd. s.yd. s.ft. s.in.; (*k*) a. sq.ch. p. sq.l.; (*l*) decl. l. dl. cl.; (*m*) gal. qt. pt. gi.; (*n*) bu. pk. qt. pt.; (*o*) d. hr. min. sec.; (*p*) cir. ° ′ ″

Illustrative Example 5 *e*.

208 ℥ 5 ʒ 1 ℈ 1 gr. $=(208\times8+5)\times(3+1)\times20+1=100161$ gr.

(For method of proof, see *Reduction Ascending*, Art. 158.)

Reduction Ascending.

157. Reduction Ascending consists in finding the number of units of a higher name which will equal a given D. N. Such questions as "How many days in 48 hours?" "How many gallons in 96 gills?" or "What compound D. N. is equivalent to 68034 ounces?" must be referred to this head. How

60)1000000 sec.=
60)6666 m.+40 sec.
24)277 hrs.+46 m.
11 d.+13 hrs.

Ans. 11 d. 13 hrs. 46 m. 40 sec.

many days in 1000000 seconds? As one would hardly remember the number of seconds in a day he would proceed by steps as illustrated by the problem solved.

158. To Reduce a D. N. to Units of a Higher Name.

Rule.—*Divide the number by as many of the given name as make one of the next greater. Setting aside the remainder, divide the first quotient by as many of its name as make one of the next greater. Continue in the same manner with the successive quotients until the required name is reached. Write the last quotient together with the several remainders for the answer.*

Each remainder is like the dividend from which it was left over.

159. Reduction Ascending and Reduction Descending mutually prove each other.

The problems under Article 156 may serve as an exercise in Ascending Reduction. See also the following:

1–21. Reduce the following to higher names—viz.: 1000 wine gal. to hhd.; 460 hrs. to ds.; 12640 far. to £; 16000 cent. to fr.; 241682 gr. to lb. Tr.; 16458 gr. to ℥; 120000 decig. to kg.; 240000 in. to fur.; 63360 in. to ch.; 4000000 sq. in. to a.; 16946195200 sq. l. to sq. m.; 40000 centiares to hect.; 50000 sq. yds. to ares; 200000 cu. in. to cu. yd.; 244.088 cu. in. to d^3, or litres, Fr.; 41683458 cu. in. to kl.; 16423 gi. to gal.; 70000 pt. to bu.; 1846387 cu. in. to bu.; 9993240 min. to yrs. of 365 d. and 6 hrs.; 72000″ to deg.

Miscellaneous Problems in Reduction.

1. How many grains in 40000 grams?

2–4. If 20 marks 45 pfennige, 25 francs 20 centimes, or $4.8665 are worth a sovereign, what sum of each currency is worth 5000 sovereigns?

5. How many metres are there in 14 miles? (Miles to in., and then in. to met.)

6. How many kl. are there in 4684 gallons?

7, 8. How many weeks in 5623480 sec.? Sec. in 12 wks.?

9. How many cu. yds. in 186324 cu. in.?

10, 11. How many sq. in. in 2 roods? In 8 sq. ch.?

12. How many lbs. av. in 10000 lbs. Tr.? ($\times 5760 \div 7000$.)

13, 14. How many bu. in 32 cu. yds.? In 40 cu. met.?

15. How many bbls. in 858568 cu. in.?

16–19. Reduce \$156 to marks; £48 to marks; 1164 fr. to marks; 1600 marks to fr.

20–22. Reduce 18000 marks to dollars U. S.; to fr.; to £.

23, 24. Reduce £150 2s. 6d. to marks; £146 10s. 4d. to marks.

25–29. Reduce to marks \$1650.72; \$1256.48; \$18645.20; \$218.45; \$364.

Fractional Denominate Numbers.

160. The same rules apply to denominate fractions, whether decimal or common, as to integers. These principles should be kept in mind—viz.:

161. (1) *The result of* **comparing two numbers** *is, at first, a* **fraction.**

162. (2) *A fractional part of any number is a* **greater** *fractional part of a* **smaller** *number.* Thus, $\frac{1}{8}$ of a gallon is 4 times $\frac{1}{8}$ of a quart.

163. (3) *A fractional part of any number is a* **smaller** *fractional part of a* **larger** *number.* Thus, $\frac{2}{3}$ of a foot is $\frac{1}{3}$ of $\frac{2}{3}$ of a yard.

164. (4) If two numbers are similar in kind, either one may be regarded as a part of the other. They should be reduced to the same name, and the divisor, which in any question is always preceded by OF, should be written under the dividend.

165. To Reduce a Fraction to a Lower Name.

RULE.—*Multiply the numerator by as many of the lower name as will make one of the name required.*

166. TO REDUCE A FRACTION TO A HIGHER NAME.

RULE.—*Multiply the denominator by as many of the lower name as will make one of the name required.*

a. 1–3. Reduce $\frac{3}{7}$ of an hour to the fraction of a day; of a week; of a year.

b. 1–4. Reduce $\frac{4}{9}$ of a foot to the fraction of an inch; of a yard; of a rod; of a furlong.

c. 1–3. Reduce $\frac{2}{9}$ of a gallon to the fraction of a pint; of a hhd.; of a quart.

d. 1–3. Reduce $\frac{2}{5}$ of a penny to the fraction of a shilling; of a £; of a quarter.

e. 1–3. Reduce $\frac{3}{4}$ of a link to the fraction of a rod; of a chain; of a mile.

f. 1–5. Reduce $\frac{642}{720}$ of a U. S. bushel to cu. in.; to wine gallons; to dry gallons; to cu. ft.; to litres.

g. 1–4. Reduce $27\frac{15}{16}$ kilolitres to wine pints; to dry gallons; to English gallons; to cubic inches.

h. 1–4. What part of an English Imp. bushel is a cu. ft.? Is a wine gal.? A litre? A dry gal.?

i. 1–4. What part of a stere is 6 wine qts.? Is 6 dry qts.? 6 Imp. or English bu.? 2 bu. U. S.?

1–33. After finding the denominate common fractions required above, reduce each to a **decimal** denominate number by dividing the numerator by the denominator. (See Article **131.**)

To illustrate the reduction of a compound D. N. to the decimal of a higher name, the following solutions are given. It will be seen that the part of the lowest name is converted into a decimal of the next higher name, and then annexed to the similar part given, making an integer and decimal—that is, a mixed number. This mixed number is then reduced to the next higher name and annexed, as a decimal, to the similar part. The result is again reduced and annexed as before, the work being continued until the required denominator is reached. Each result includes all the preceding small parts.

What decimal of a wk. is 5 d. 12 h. 30 m. 36 sec.?

60)36	sec.
60)30.6	m.
24)12.51	h.
7) 5.52125	d.
.78875	wk.

Analysis.—In any period of time there are $\frac{1}{60}$ as many minutes as there are seconds. As $\frac{1}{60}$ of 36=.6, therefore 36 sec.=.6 m. There are $\frac{1}{60}$ as many hours as there are minutes, therefore 30.6 m.=.51 h. There are $\frac{1}{24}$ as many days as there are h., therefore 12.51 h.=.52125 d. There are $\frac{1}{7}$ as many weeks as there are days, therefore 5.52125 d.= .78875 weeks.

What decimal of a fur. is 10 r. 3 yd. 2 ft. 3 in.?

12)3	in.
3)2.25	ft.
$5\frac{1}{2}$)3.75	yd.
40)10.5909	r.
.264775	fur.

Three in. equal $\frac{1}{12}$ as many ft., or .25 ft., which, with 2 ft., makes 2.25 ft. There are $\frac{1}{3}$ as many yds. as ft., therefore 2.25 ft. make .75 yd., which, with 3 yd., make 3.75 yd. As 11 half-yards make a rod, 3.75 yd. must be divided by $\frac{11}{2}$ if it is to be reduced to the decimal of a rod. This decimal, with 10 rods, divided by 40, will be reduced to the decimal of a furlong, making in all .264775 fur.

What decimal of a kilogram is 8 dr. 8 oz. 1 lb.?

16)8	dr.
16)8.5	oz.
2.2046)1.53125	lb.
.69457	kg.

$\frac{1}{16}$ of 8, the number of oz., = .5, which, added to 8 oz.=8.5 oz. 8.5 oz. reduced = .53125 lb. This, annexed to 1 lb., makes 1.53125 lb. This number divided by the number of lbs. in a kilogram will make a quotient representing kilograms 1.53125÷2.2046=.69457 kg.

What decimal of 5 cu. ft. 1000 cu. in. equals 2 bu. 3 pks.?

4)3 pks.

2.75 bu.=

.2.75×2150.4 cu. in.=5913.6 cu. in.

In 5 cu. ft. 1000 in. there are 9640 cu. in.

5913.6 cu. in.=.613444 of 9640 cu. in.

What decimal of a cu. metre is a bbl.?

1 bbl.$=31\frac{1}{2}\times231$ cu. in., or 7276.5 cu. in.

1 cu. metre$=61022$ cu. in.

7276.5 cu. in.$=.1192$ of a cu. metre.

PERCENTAGE.

Percentage relates to estimates based on a **hundred.**

320.* The result obtained by taking one or more **hundredths** of a number is called the **percentage.**

321. That number of which the hundredths are taken is called the **base.**

322. The number of hundredths taken is called the **rate.**

323. 2 hundredths of 800 is 16. In this statement 800 is the *base*, 2 is the *rate*, and 16 is the *percentage.*

324. The phrase *per cent.* is an abbreviation of the Latin expression, *per centum*, by the hundred. Its sign is %. Two hundredths of 800 may be written 2% of 800. As all common fractions may be expressed in hundredths, any part of anything considered as a basis of a calculation may be treated as a certain per cent. of this base. Thus $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{20}$, $\frac{1}{25}$, or $\frac{1}{50}$ of a number or quantity may be written as 50%, 25%, 20%, 10%, 5%, 4%, or 2% of the given quantity. When one number is considered as a part of another, the *part* must be thought of as a *numerator*, and the measure, standard, or *base*, as a *denominator* of a fraction. Thus, in the question, "What part of 8 is 2?" the result is $\frac{2}{8}$ or $\frac{1}{4}$. This fraction equals $\frac{25}{100}$. It follows, then, that if one number may be considered as a

*** The "Standard Arithmetic" Number One concludes with Art. 319. In view of the fact that many pupils will have completed that volume before taking up this, the succeeding articles have been numbered to correspond with those of the first book.**

part of another, that the same number may be considered as a certain per cent. of that other.

325. The business terms **principal, cost, policy, valuation,** etc., are names for ***base,*** and the terms corresponding to these, viz.: **interest, profit** or **loss, insurance, tax,** are names for ***percentage.***

326. The annual **interest** on any sum of money equals a fraction of the principal. This fraction, reduced to *hundredths,* is called the ***rate.***

327. A **tax** is a fraction of the **valuation** of a piece of property; this fraction reduced to *hundredths* is called the ***rate.***

\$12 is 6% of \$200. The percentage 12 is the number representing the part. The per cent. is the fraction made by taking this number as a dividend together with the base as a divisor. Thus, 12 taken in this way with 200 makes $\frac{12}{200}$ or $\frac{6}{100}$ or 6%. When the rate is 6%, the interest on \$200 will be \$12; so, at the same rate, the tax on \$200 will be \$12.

328. Any number when regarded as hundredths of another number expresses percentage.

Any common fraction may be reduced to hundredths (Article 130). Pupils should become familiar with the following

TABLE.

$\frac{1}{100}$	= .01	= 1%	$\frac{3}{4}$	= .75	= 75%
$\frac{1}{50}$	= .02	= 2%	1	= 1.00	= 100%
$\frac{1}{25}$	= .04	= 4%	$2\frac{1}{2}$	= 2.50	= 250%
$\frac{1}{20}$	= .05	= 5%	$\frac{1}{8}$	= .125	= $12\frac{1}{2}$%
$\frac{1}{10}$	= .10	= 10%	$\frac{1}{6}$	= $.16\frac{2}{3}$	= $16\frac{2}{3}$%
$\frac{1}{5}$	= .20	= 20%	$\frac{1}{12}$	= $.08\frac{1}{3}$	= $8\frac{1}{3}$%
$\frac{1}{4}$	= .25	= 25%	$\frac{1}{200}$	= .005	= $\frac{1}{2}$%
$\frac{1}{2}$	= .50	= 50%	$\frac{1}{800}$	= .00125	= $\frac{1}{8}$%

Oral Exercise.

What per cent. of $100 are $2? $5?

What per cent. of a dozen pens are 3 pens?

How many hundredths of 100 is 2? 6? 9?

What per cent. of 4 is 1? Is 2? Is 3?

What per cent. of 20 is 3? Is 4? Is 5?

What per cent. of 25 is 6? Is 7? Is 8?

What per cent. of 10 is 3? Is 9?

How many hundredths is $\frac{1}{50}$? $\frac{1}{10}$? $\frac{1}{20}$? $\frac{1}{25}$? $\frac{1}{5}$?

Reduce $\frac{3}{50}$ to hundredths; $\frac{4}{50}$; $\frac{7}{10}$; $\frac{3}{5}$; $\frac{3}{4}$; $\frac{2}{25}$.

How many hundredths of 24 is 6? Of 12 is 3?

How many hundredths of 10 is 2? Of 10 is 4? 6? 9?

What per cent. of 16 is 8? Of 16 is 4? 12?

What % of 20 is 2? Of 20 is 6? 12? 18?

Name the *base* in each of the following: 16 is 25% of 64; 32 is 50% of 64; 48 is 75% of 64; 8 is 12½% of 64; 24 is 37% of 64.

Name the *rate* in each. Name the *percentage* in each?

What common fraction equals 25%? 30%? 2%? 10%? 12½%? 16⅔%? ½%? ⅛%?

What decimal will express 2½%? 3¼%? 1⅛%? ½%? 6%? 7½%?

What is 1% of 100? *Ans.* 1.

What is 4% of 100? Of 200? Of 250?

329. *To find the Percentage.*

Rule.—*Multiply the base by the rate.*

Exercise XCIX.

1. A drover sold 25% of a drove of 420 cattle. How many cattle did he sell?

2. How many eggs were lost, if in a cask containing 90 dozen 4% were broken?

3. If the death-rate in a city of 120,000 inhabitants be 2½% every year, how many of its people die annually?

4. How many persons die if the mortality is 3% in a population of 500,000?

5. If a man who agreed to supply 10,000 lbs. of butter has furnished 55% of it, how many lbs. has he delivered?

6. The expenses of a certain establishment which, up to a certain time, were \$480 a week, were reduced $12\frac{1}{2}$%. What was the weekly reduction?

7. If in a barrel containing 640 apples $33\frac{1}{3}$% have decayed, how many apples remain sound?

8. What are the items in the expenditure of \$1,200, if 8% of it be spent for books, 10% for clothes, 12% for fares, 20% for rent, 30% for food, and the remainder for incidentals?

9. If a clerk whose income is \$2,400 should save 8% of it one year, 18% the next year, 28% the third year, and 25% the fourth year, what would he accumulate in the four years?

10. A boy, who had \$120, expended 10% of it; he then laid out 12% of the remainder; and, finally, spent 50% of what was left. What did he disburse in all?

330. *To find the Rate.*

RULE.—*Divide the percentage by the base.*

Let it be required to find how many hundredths of 25 is 1. 1 is $\frac{1}{25}$ of 25. $\frac{1}{25}=\frac{4}{100}$. 1 is therefore $\frac{4}{100}$ of 25.

If the question were, "What per cent. of 64 is 48?" the answer would be the result of reducing $\frac{48}{64}$ to hundredths. $\frac{48}{64}=\frac{3}{4}$, or .75.

EXERCISE C.

What per cent.

1. Of 72 is 18?
2. Of 750 is 17.5?
3. Of 812 is 640?
4. Of 350 is 150?
5. Of 98 is 36?
6. Of 412 is 25?
7. Of 250 is 10?
8. Of \$1,200 are \$60?
9. Of \$464 are \$16?
10. Of £36 are £9?

What per cent.

11. Of 18 gross are 6 dozen?
12. Of 200 days are 12 days?
13. Of 300 lbs. are 20 lbs.?
14. Of 20 lbs. are 12 oz.?
15. Of 30 gals. are 3 gals.?
16. Of 40 gals. are 2 pts.?
17. Of 4 da. 10 hrs. are 3 da.?
18. Of 3 y. 2 mo. are 2 mo.?
19. Of 5 y. 3 mo. are 2 y. 1 mo.?
20. Of 10 bu. 2 pks. are 3 bu.?

331. *To find the Base.*

RULE.—*Divide the percentage by the rate.*

Let it be required to find what number 12 is 2% of.

If 12 is 2 hundredths, one-half of 12, or 6, is 1 hundredth. If 6 is 1 hundredth of a number, 100 hundredths is 100 times 6 or 600. The result of this analysis equals the quotient of 12 divided by the given rate; for $12 \div \frac{2}{100} = 600$.

Again: What is that number of which 24 is 6%?

SOLUTION.—If 24 be 6%, $\frac{1}{6}$ of 24, or 4, is 1%; and 100%, or the whole of the number, is 100 times 4 or 400. Now, $24 \div .06 = 400$.

EXERCISE CI.

Of what number	Of what are
1. Is 12 2%?	11. 12 bu. 25%?
2. Is 96 12%?	12. 18 gal. 20%?
3. Is 360 20%?	13. $25 $12\frac{1}{2}$%?
4. Is 200 $33\frac{1}{3}$%?	14. 36 men $33\frac{1}{3}$%?
5. Is 378 42%?	15. 48 days 50%?
6. Is 480 120%?	16. 29 rods 4%?
7. Is 900 75%?	17. 33 hours $8\frac{1}{2}$%?
8. Is 5 $\frac{1}{2}$%?	18. 42 y. 5%?
9. Is $22\frac{1}{2}$ $2\frac{1}{2}$%?	19. 56 dwt. 22%?
10. Is 60 $16\frac{2}{3}$%?	20. 60 drams 44%?

These rules for percentage may be applied to the computation of Interest, Discount, Profit or Loss, Commission, Insurance, Taxes, and Duties.

INTEREST.

332. **Interest** is money paid for the use of money.

333. The money lent is called the **Principal.**

334. The interest allowed **annually** on one dollar is computed at a certain **Rate**, and equals a fraction of the principal.

335. The sum of the Principal and Interest is called the **Amount.**

336. ***The Interest*** **varies** as the **Principal** is more or less than 100, and as the **Time** is greater or less than one year, and also as the **Rate** is greater or less than 6%. A person may hire a large or a small sum of money for a longer or shorter time at 3%, 4%, 5%, 6%, or 7%, or at any rate which he may choose to pay. *In brief, the interest will depend on the principal, on the rate, and also on the time.*

337. If a man hires the use of a dollar for a year and agrees to pay at the rate of 6%, the interest will be $\frac{6}{100}$ of the principal, or 6 cents. It will be seen that the *number of cents* paid is equal to *half the number of months* in a year; that is, he pays 6 cents for 12 months, or 1 cent for 2 months.

If he should hire the dollar for 2 months, he would pay 1 cent. If he should hire $100 for 60 days, he would pay $1; if $200, then the interest would be $2; if $450, $4.50; $96.10, $.96$\frac{1}{10}$.

338. To find the Interest on any Sum for 60 Days at the Rate of 6% per Year.

RULE.—*Point off two places from the right of the principal.*

Exercise CII.

Required, the interest at 6% for 60 days of the following: $800; $450; $973; $1,005; $1,050; $44.16; $780.42; $963.09; $49.98; $961.92; $827?

What is the interest on each of the above sums for 4 times 60 days? For 3 times 60 days? For 1½ times 60 days (*i.e.*, for 90 days)? For 2¼ times 60 days (135 days)? For 30 days? For 15 days? For 20 days?

NOTE.—The third decimal figure stands for *mills.* If there be less than 5 mills, in the result they are usually rejected; if more than five, they are reckoned as a cent.

Recapitulation.

339. Interest at 6% for 60 days may be instantly shown by pointing off the *two* right-hand figures of the principal. If interest is required for more or less time, this result may be increased or diminished according to the conditions; and, if the given *rate* be more or less than 6%, the last result may be altered to meet the case.

Let it be required to find the interest on $1,050 for 93 days at 7%.

$10.50	=	interest for	60 days		@ 6%.	
5.25	=	"	"	30	"	
.525	=	"	"	3	"	
6)16.275	=	"	"	93	"	
2.712	=	"	"	"	"	@ 1%.
$18.99	=	"	"	"	"	@ 7%.

340. General Rule.—*Point off the two right-hand figures of the principal, and the result will be the interest for* 2 *months, or* 60 *days, at* 6%. *By means of multiples or aliquot parts of this interest taken for periods greater or less than* 60 *days, find the interest at* 6% *for the entire time. Add or subtract such aliquot parts of the interest at* 6% *as may be required by the given rate.*

341. Note.—If the time is stated in years or months, multiply the interest for 60 days by one-half the number of months. If interest for days be required, multiply one-tenth of the interest for 60 days by one-sixth of the number of days—or find such aliquot parts as will, when taken together, represent the interest for the total number of days. See the following illustrative examples:

a. Req. interest for 3 yrs. 6 mos., at 8%, on $1,260. 3 yrs. and 6 mos.$=21\times2$ mos.; and $8\%=\frac{4}{3}$ of 6%. $12.60\times21\times\frac{4}{3}=$(by cancellation) 12.60×28 or $352.80.

b. Int. on $1,680.60, for 4 yrs. 8 mos. 12 ds., at $7\%=c$.

c. $\$16.8060 \times 28.2 + \dfrac{\$16.806 \times 28.2}{6} = \$540.96 + \$90.16 =$
$631.12.

Exercise. CIII.

	a.		*b.*		*c.*
1.	$72	@	4 %	for	93 days.
2.	$125	@	5 %	"	123 days.
3.	$1293	@	6 %	"	1 yr. 4 mos. 7 days.
4.	$421.18	@	$7\frac{1}{2}$%	"	2 yrs. 3 mos. 18 days.
5.	$1395.40	@	$9\frac{1}{2}$%	"	1 yr. 5 mos. 9 days.
6.	$12000	@	$4\frac{1}{2}$%	"	3 yrs. 6 mos.
7.	$175.42	@	$5\frac{1}{2}$%	"	1 yr. 1 mo. 12 days.
8.	$296.04	@	7 %	"	5 yrs. 7 mos. 12 days.
9.	$1872.00	@	7 %	"	6 mos. 3 days.
10.	$262.75	@	$6\frac{1}{2}$%	"	4 mos. 3 days.

Required the interest on each of the above sums at every one of the given rates for the time set opposite each principal. (100 problems. The first ten results given will refer to the first principal, the next ten results to the second principal, viz. : $125, etc.)

Interest by Aliquot Parts.

342. To compute interest for months and days, a familiar acquaintance with the following tables will be found useful :

Of 1 yr or 12 mos.		Of 1 mo. or 30 days.	
2 months	$=\frac{1}{6}$.	2 days	$=\frac{1}{15}$.
3 "	$=\frac{1}{4}$.	4 "	$=\frac{1}{10}+\frac{1}{3}$ of $\frac{1}{10}$.
4 "	$=\frac{1}{3}$.	7 "	$=\frac{1}{5}+\frac{1}{6}$ of $\frac{1}{5}$.
5 "	$=\frac{1}{4}+\frac{1}{6}$.	8 "	$=\frac{1}{6}+\frac{1}{10}$.
5 "	$=\frac{1}{3}+\frac{1}{4}$ of $\frac{1}{3}$.	9 "	$=\frac{1}{5}+\frac{1}{10}$.
6 "	$=\frac{1}{2}$.	11 "	$=\frac{1}{5}+\frac{1}{6}$.
7 "	$=\frac{1}{2}+\frac{1}{6}$ of $\frac{1}{2}$.	12 "	$=\frac{1}{5}+\frac{1}{5}$.
7 "	$=\frac{1}{3}+\frac{1}{4}$.	13 "	$=\frac{1}{3}+\frac{1}{10}$.
8 "	$=\frac{1}{2}+\frac{1}{3}$ of $\frac{1}{2}$.	14 "	$=\frac{1}{5}+\frac{1}{6}+\frac{1}{10}$.
8 "	$=\frac{1}{3}+\frac{1}{3}$.	16 "	$=\frac{1}{3}+\frac{1}{5}$.
9 "	$=\frac{1}{2}+\frac{1}{4}$.	17 "	$=\frac{1}{5}+\frac{1}{5}+\frac{1}{6}$.
9 "	$=\frac{1}{2}+\frac{1}{2}$ of $\frac{1}{2}$.	18 "	$=\frac{1}{2}+\frac{1}{10}$.
10 "	$=\frac{1}{2}+\frac{1}{3}$.	19 "	$=\frac{1}{3}+\frac{1}{6}+\frac{1}{10}$.
11 "	$=\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$.	20 "	$=\frac{1}{3}+\frac{1}{3}$.

a. What is the interest on $528 for 1 yr. 7 m. 23 d.?

It is the sum of the interest for these separate periods, viz., for 1 yr. + 6 mo. + 1 mo. +15 d.+ 5 d.+3 d., or $60.88. The fractions at the left represent the relative periods for which the interest on the principal is to be reckoned. The sums of interest will depend on the relative periods of time. Thus, the interest for 6 months will be ½ the interest for 1 year; for 1 month it will equal $\frac{1}{6}$ of 6 months' interest; for 15 days, ½ of 1 month's interest.

	$528	
	.07	
½	$36.96	= Int. for 1 yr.
$\frac{1}{6}$	18.48	= " " 6 mo.
½	3.08	= " " 1 "
$\frac{1}{3}$	1.54	= " " 15 d.
$\frac{1}{5}$	.513	= " " 5 d.
	.308	= " " 3 d.
	$60.88	

b. What is the interest on $564.58 for 3 years 8 months and 12 days, at 6 per cent.?

		$564.58	
		.06	
6 months	½ year	33.8748	Interest for 1 year.
		3	
		101.6244	" " 3 years.
2 months	$\frac{1}{3}$ of 6 mo.	16.9374	" " 6 months.
12 days	$\frac{1}{5}$ of 2 mo.	5.6458	" " 2 months.
		1.1291	" " 12 days.
		$125.34	" " 3 y. 6 mo. 12 d.

c. Required int. $180.50 for 2 yr. 11 mo. 19 d.

		$180.50	
		.07	
		12.6350	
		2	
		25.27	
8	$\frac{1}{3}$	8.42	Interest for 8 mo. being $\frac{1}{3}$ of that for 2 years.
3	$\frac{1}{8}$	3.16	" " 3 " " $\frac{1}{8}$ " for 2 "
10	$\frac{1}{9}$	.351	" " 10 d. " $\frac{1}{9}$ 90 days' interest.
9	$\frac{1}{10}$	.316	" " 9 d. " $\frac{1}{10}$ " " "
		$37.52	

Exercise CIV.

Required, the interest in the following examples:

	Principal.		Rate.	Time.
1.	\$453.45	@	6%	for 3 yr. 4 mo.
2.	\$225.375	@	6%	" 4 yr. 7 mo.
3.	\$1436.59	@	7%	" 2 yr. 5 mo. 18 d.
4.	\$29.08	@	7%	" 5 yr. 8 mo. 20 d.
5.	\$526.148	@	$6\frac{1}{2}$%	" 6 yr. 11 mo.
6.	\$1000.	@	8%	" 5 yr. 9 mo. 13 d.
7.	\$48.25	@	5%	" 1 yr. 2 mo. 17 d.
8.	\$7500.	@	7%	" 4 mo.
9.	\$243.81	@	$7\frac{1}{2}$%	" 7 mo. 25 d.
10.	\$101.30	@	6%	" 10 mo. 4 d.
11.	\$224.14	@	7%	" 8 mo. 13 d.
12.	\$438.24	@	7%	" 4 yr. 9 mo. 14 d.
13.	\$140.50	@	9%	" 4 yr. 6 mo. 15 d.
14.	\$325.25	@	$6\frac{1}{2}$%	" 2 yr. 9 mo. 12 d.
15.	\$495.95	@	$6\frac{3}{4}$%	" 5 yr. 5 mo. 5 d.
16.	\$436.75	@	$3\frac{4}{5}$%	" 7 yr. 9 mo. 18 d.
17.	\$2560.75	@	$6\frac{1}{3}$%	" 4 yr. 3 mo. 25 d.
18.	\$1112.10	@	$5\frac{1}{6}$%	" 2 yr. 2 mo. 2 d.
19.	\$888.88	@	$5\frac{1}{4}$%	" 1 yr. 11 mo. 10 d.
20.	\$946.35	@	$10\frac{1}{2}$%	" 1 yr. 4 mo. 15 d.

343. Accurate Interest (N. Y. Law).

Rule.—*Divide the product of the principal, the rate, and the exact number of days by* 365.

What is the interest on a note of \$720 drawn January 1, 1876, payable May 3, 1876, at 7%?

Days $= 31+29+31+30+2 = 123$. \$720 $\times$ .07 $\times$ 123 = \$6199.20. \$6199.20 $\div$ 365 = \$16.96

Exercise CV.

Required, accurate interest in the following problems:

1. \$1275.46 at (*a*) .07, from Mar. 1, 1873, to Mar. 12, 1874.

2. \$642.75 at (*b*) .08, from Apr. 1, 1872, to Jan. 13, 1875.
3. \$10049.72 at (*c*) .06, from May 1, 1874, to July 29, 1875.
4. \$8479.35 at (*d*) .05½, from Jan. 1, 1871, to Jun. 1, 1874.
5. \$450.75 at (*e*) .07½, from Feb. 4, 1864, to May 9, 1865.
6. \$1890.34 at (*f*) .03½, from Apr. 10, 1865, to Jun. 16, 1865.
7. \$1200.00 at (*g*) .06½, from Jun. 12, 1866, to Jan. 19, 1867.
8. \$9684.27 at (*h*) .09, from Dec. 15, 1872, to Feb. 20, 1876.
9. \$972.00 at (*i*) .10, from Feb. 11, 1876, to Aug. 1, 1876.
10. \$164.50 at (*j*) .12, from Jan. 12, 1876, to Dec. 12, 1876.

NOTE.—The above exercise will afford 100 problems, if the interest on each principal be found at every one of the rates given.

Suggestion.—Arrange the pupils in sections of ten, and number each individual of a section. Assign problems as follows :

There are ten rates lettered. The pupils of Section One compute the interest on \$1275.46, each at the rate corresponding to his number; the pupils of Section Two compute the interest on \$642.75 in like manner ; etc.

344. TO FIND EITHER THE PRINCIPAL, RATE, OR TIME.

The product of the principal (p), rate (r), and time (t), equals the interest (i). In brief, $p\times r\times t=i$. By the ordinary principles of multiplication and division we may solve such problems as follow, thus :

a.	$2\times3\times5=?$	In a $?=30$
b.	$?\times3\times5=30$	In b, $?=\frac{30}{15}$
c.	$2\times?\times5=30$	In c, $?=\frac{30}{10}$
d.	$2\times3\times?=30$	In d, $?=\frac{30}{6}$.

And in general, *the product of three factors divided by the product of any two of these three, will equal the other factor.*

Now, as $p \times r \times t = i$, \$24 @ 6% for 2 yrs. $i = \$2.88$.

$$p = \frac{i}{r \times t} \quad . \quad . \quad . \quad \$24 = \frac{2.88}{.06 \times 2} = \$24.$$

$$r = \frac{i}{p \times t} \quad . \quad . \quad . \quad .06 = \frac{2.88}{\$24 \times 2} = .06.$$

$$t = \frac{i}{p \times r} \quad . \quad . \quad . \quad 2 = \frac{2.88}{24 \times .06} = 2.$$

Exercise CVI.

1–5. In what time will \$424 yield **\$72** int. at 6% ; at 7% ; at $7\frac{1}{2}$% ; at 4% ; at 10% ?

6–10. In what time will it yield **\$80** interest at each of the rates mentioned ?

11–50. In what time will it yield **\$34** ; **\$12** ; **\$122** ; **\$14.60** ; **\$7** ; **\$40.50** ; **\$73** ; \$424 ?

N.B.—First pupil take interest \$72, and rate 6% ; second pupil, interest \$80, rate 6% ; third pupil, interest \$34, rate 6% ; fourth pupil, interest \$12, rate 6% ; fifth pupil, interest \$122, rate 6%.

The directions just given will apply to sections of five pupils. In sections of 10, the first may take the 1st interest and the 1st rate ; the second pupil, the 2d interest and the 1st rate ; etc., etc.

345. To find the *Time*, the Principal, Rate, and Interest being given.

Rule.—*Divide the given interest by the interest for one year.*

In what time will \$360 amount to \$473.40 at 7% ? \$473.40 —\$360 = \$113.40.

The interest on \$360 for 1 yr. at 7% is \$25.20.

\$113.40 ÷ \$25.20 = $4\frac{1}{2}$ or 4 yrs. 6 mos.

In what time will \$264 yield \$27 interest at 7%?
" " " " \$1256 " \$180 " " $6\frac{1}{2}$%?
" " " " \$428 " \$9.60 " " 6%?
" " " " \$1000 " \$1000 " " 7%?

346. To find the *Rate.*

Rule.—*Divide the given interest by the interest at 1% for the specified time.*

347. To find the *Principal.*

Rule.—*Divide the given interest by the interest on \$1, for the specified rate and time.*

348. To find what Principal will amount to a given sum at a specified Rate and Time.

Rule.—*Divide the amount given by the amount of \$1 at the rate specified, and for the given time.*

Note.—To economize space, the abbreviations P., R., T., and A. will be used for the words principal, rate, time, and amount when they would occur in the following questions:

Exercise CVII.

Required the R.:

P.	T.	A.
\$1260.	2 yr. 6 m.	\$1449.
1200.	4 yr.	1464.
64.50	$3\frac{1}{2}$ yr.	82.56
1350.	5 yr.	1687.50
750.50	8 yr.	1020.68
100.	9 m. 18 d.	104.80
1740.	3 yr.	2175.
196.	5 yr. 7 m.	294.49
120.	1 yr. 6 m. 20 d.	$133.06\frac{2}{3}$
575.50	1 yr. 4 m. 20 d.	623.46

Exercise CVIII.

Required the P.:

T.	R.	A.
3 yr.	6%	\$500.
1 yr. 2 m.	5%	340.
2 yr. 6 m. 24 d.	6%	46.16
1 yr. 10 m. 15 d.	6%	1200.
1 yr. 2 m. 12 d.	6%	128.64
2 yr. 3 m. 15 d.	9%	410.73
1 yr. 9 m. 3 d.	10%	1176.13
3 m. 3 d.	12%	25.78
1 yr. 11 m. 29 d.	7%	165.50
3 yr. 2 m. 21 d.	8%	566.10

Bank Discount.

349. ***A Promissory Note*** is a written promise by one party to pay to another a definite sum of money on demand or at a certain time. The promisor is called the **maker** or principal, the promisee is called the **payee**, and the person who owns the note is called the **holder**. The payee of a note made to "order" is also the **endorser**. The sum mentioned in the note is called its **face**.

$75. UTICA, Jan. 1, 1876.

Three months after date I promise to pay to George Smith, or order, seventy-five dollars. Value received.

(Due April 4, 1876.) THOMAS FREELAND.

In this note, Thomas Freeland is the *maker* or drawer; George Smith, the *payee;* George Smith, by writing his name across the back of the note, becomes the *endorser*, and the person to whom he then transfers it becomes the *endorsee* or holder.

$200. NEW YORK, June 10, 1876.

On demand, for value received, I promise to pay to A. B., or bearer, two hundred dollars. C. D.

350. When a note is made payable to A. B., or order, or bearer, it is a **negotiable note**; that is, it is transferable by assignment or endorsment to another person, the drawer being obliged to pay to the legal holder the sum specified when it falls due. When the note falls due, the holder must first promptly demand payment of the maker, and, if he refuses, the holder may demand payment of A. B., the endorser.

351. A note will draw interest from the time it becomes due, though no mention of interest is made. If made payable on demand, it will draw interest from the time the demand is made.

352*a*. ***A Bank*** is a chartered corporation authorized to issue bank-notes for circulation, to receive deposits, to discount notes, to buy and sell exchange, etc.

352*b*. *Bank Discount* is interest on the entire principal deducted in advance. Among commercial men, when money is lent on a note, interest is first reckoned on the face of the note for three days more than the specified time, and, this being deducted from the face of the note, the remainder is paid over to the borrower. The interest charged on notes discounted at a bank is computed on the amount due on the note at maturity; that is, at the time when the note falls due. In most States, banks are allowed by law to charge bank discount for periods of one year or less.

352*c*. If a bank discount A.'s note drawn for $100, and payable 3 months after date, A. will receive but $98.19. Three months and three days later he must deposit $100 to meet the note.

352*d*. The bank discount is reckoned as if it were *simple interest on the face of the note for 3 days more than the specified time.*

352*e*. The face of the note, less the bank discount, is called the **Proceeds.** In the last instance, the bank discount is $1.81, and the proceeds $98.19. The true discount is but $1.78. At **true discount** a borrower pays interest *on what he really borrows.* At bank discount, the rate being 7%, a person giving a note drawn payable 14$\frac{2}{7}$ years after date *would receive no proceeds,* as the bank discount would equal the face of the note.

1. Required, the proceeds of the following note, discounted on the day of its date:

$900. New York, May 1, 1877.

Four months after date, for value received, I promise to pay Edward Rowe, or order, nine hundred dollars.

(Due Sept. 4, 1877.) E. J. Whitlock.

The legal rate in New York is 7%. The interest on $900 for 4 mo. 3 da. is $21.53. This deducted from $900 equals $478.47, or the proceeds.

2. Required, the proceeds of the following note:

$900. NEW YORK, May 1, 1877.

Ninety days after date I promise to pay William Wood, or order, $900, with interest. at the Park National Bank. Value received. SMITH ELY, JR.

(Due Aug. 2, 1877.)

As this note is drawn payable with interest, and as in New York each day's interest must be only $\frac{1}{365}$ of the year's interest, the amount payable at its maturity is $916.05. The bank discount on this for 93 days (by rule for accurate interest) is $16.34. The proceeds are $899.71.

Usury is unlawful interest. To subject a lender to the penalty prescribed by law he must be proved to have *intentionally* charged or taken more than the legal rate of interest. The taking of interest in advance by banks or by individuals, on commercial paper, such as notes or drafts, having but a short time to run is not usurious.

EXERCISE CIX.

Find the proceeds of the notes described as follows:

	Face of Note.	Date.	Time when due.	Rate.	
1, 2.	$1250.50	April 1, 1874.	May 31, 1874.	6%	7%
3, 4.	1000.	Jan. 4, 1874.	Apr. 3, 1874.	"	"
5, 6.	706.20	Jan. 10, 1874.	Apr. 1, 1874.	"	"
7, 8.	808.76	Jun. 10, 1874.	Dec. 12, 1874.	"	"
9, 10.	438.60	Nov. 12, 1875.	Jan. 15, 1876.	"	"
11, 12.	500.00	Sept. 3, 1874.	Jan. 3, 1875.	"	"
13, 14.	230.00	Dec. 3, 1874.	Feb. 25, 1875.	"	"
15, 16.	1521.70	Nov. 9, 1873.	Dec. 29, 1873.	"	"
17, 18.	2060.	Nov. 9, 1873.	Feb. 8, 1874.	"	"
19, 20.	1500.	Nov. 9, 1873.	Jan. 8, 1874.	"	"

Reckon 360 days to the year.

352*f*. If it were required to draw a note for 60 days which,

being discounted at 6%, should yield \$1000, it would be necessary to find what a note drawn for \$1 would yield as proceeds. The interest on \$1, at 63 days for 6%, is \$.0105, hence the proceeds are \$.9895. Now, just as many times as \$.9895 are wanted, just so many times must \$1 be written as the face of the note. \$1000=\$.9895×1016.61; or, in another form, \$1000 ÷.9895=\$1016.61.

To find the Sum for which a Note must be drawn to yield any required Proceeds.

Rule.—*From* \$1 *deduct the interest on* \$1, *computed at the given rate and for the given time, plus* 3 *days, and divide the required proceeds by the remainder.*

Exercise CIX. a.

1–20. Consider each sum mentioned in Exercise CIX. as proceeds required, and find the several amounts for which the notes should be drawn. Thus, in problem 1, the proceeds being \$1250.50, the face of the note would equal \$1250.50÷.9945. or \$1257.42.

True Discount.

353. ***Discount*** is interest paid in advance. The sum which would amount to a debt due at a certain future time would be the **present worth** of the said debt.

354. The true discount on a note in which the words "with interest" are omitted, is such a sum as will amount to the face of the note.

355. If \$1.01 is to fall due 60 days hence, interest being estimated at 6%, then \$1, if paid now, will discharge the debt; for in 60 days at 6% \$1 will amount to \$1.01. The discount in this case is 1 cent. If the prospective debt were \$101, then the discount would, of course, be \$1, and the present worth would be \$100. If the debt were \$450, the number of times

$1.01 which that amount represents will equal the number of dollars which might be paid at the present time to cancel the debt. $450÷1.01=$445.54, or the P. W., and $4.46=T. D.

To find the Present Worth.

Rule.—*Divide the prospective debt by the amount of* $1.

356. True Discount *equals the prospective debt or face of the note minus the present worth.* In the following exercise compute at 7%, unless some other rate is specified. In New York consider fractions of a month as days and reckon such days as 365ths of a year.

Exercise CX.

1. What is the present worth of a note for $350 due 4 months hence, money being worth 8% ?

2. If a person who has stipulated to pay $900 on the 1st of Sept. should oblige his creditor by paying it on the 1st of March preceding, what discount should be allowed ?

3. If a person should lend B. money and receive B.'s note for $750, to be paid in 4 months, no interest being expressed, how much would B. have borrowed on that note ?

4. If a man discounts his own note for $400, payable 3 months later, what is the present worth ?

5. A note drawn June 4, 1875, for $1250 and payable 9 months after date, was paid Sept. 10, 1875. What sum was required ?

6. What was the discount on a note drawn for $824.62, due March 10, 1876, the maker having paid it Dec. 4, 1875.

7. If a man buy 8 shawls at $200 each, agreeing to pay for them in 3 months, and should, after keeping them 15 days, sell them at $212.50, receiving a note due in 4 months, what would he have gained if he had discounted the note at the time the first debt fell due; at the rate of 7%. *Ans.* $85.25.

7*b*. What would have been his gain if he had gotten the note discounted at a bank at 6% ?

This latter problem must be solved as follows: By the conditions, a note drawn for \$1700 is presented at a bank for discount $1\frac{1}{2}$ months before it falls due. Banks charge interest on more money than they actually lend. In this case they would charge interest on \$1700, at 6%, for $1\frac{1}{2}$ months. Deducting this interest, \$12.75, from \$1700, there would be paid to the merchant \$1687.25. As the goods actually cost \$1600, the gain is \$87.25.

Exercise CXI.

Find the proceeds of the following, remembering to add 3 days to the time specified in each case:

Face of Note.	Time.	Rate.	Face of Note.	Time.	Rate.
1. \$750.	60 days	6%	6. \$350.25	72 days	9%
2. 125.	90 days	7%	7. 845.32	12 days	12%
3. 476.	30 days	8%	8. 1000.	19 days	10%
4. 1475.	4 mos.	5%	9. 1200.	40 days	4%
5. 835.50	6 mos.	$5\frac{1}{2}$%	10. 1698.	50 days	3%

If the discount on each note were computed at *all* the *rates*, 100 problems would be solved; and, if it were proposed to find the discount on each note at every separate *rate*, and separate *time*, 1,000 problems would be stated.

357. *Commercial Discount*

Is a **deduction** made from the face of a note or from a stipulated price **without respect to time.** Thus, the price of goods may have been stated with the understanding that credit should be given. The deduction made if cash is paid is generally more than would equal the interest for the contemplated time.

If a man buys a bill of goods amounting to \$1000 on a credit of 60 days with the option of 5% for cash, he might, by borrowing the money at 6%, save \$40, for the interest he would pay would be but \$10, while the commercial discount would be \$50.

The difference between a wholesale and a retail price is often called the discount. It is allowed to dealers who buy to sell again.

Exercise CXII.

1–21. Find the commercial discount on each of the following at $2\frac{1}{2}$%, 25%, and $33\frac{1}{3}$% off, viz., 3 results to each: $150; $754; $125.50; $62; $96; $110; $84.

22. What were the net proceeds of an invoice of stationery amounting to $840, retail, the wholesale discount being 25%, if an allowance of $2\frac{1}{2}$% was made for cash?

23. What would a buyer gain who should cash his own note, given for $1400 at 4 months, by borrowing the money at bank discount, 7%, if by so doing he should secure a commercial discount of 10%?

24. What is the difference between a discount of 25% and a discount of 20% off, and then 10% off the remainder, the bill being $500.

Interest Notes.

358. If, instead of cash payments of interest falling due, notes be given, the entire interest on the original debt and on the unpaid interest may be computed as follows:

Rule.—*Find the sum of the several periods of time for which the notes draw interest, and add the interest on one note for this total period to the interest on the original principal.*

Each note, as well as the principal, draws simple interest till paid.

Example.—A note for $3000 was given January 1, 1868, with interest at 6%, payable semi-annually. Notes, instead of cash, were given for the semi-annual interest as it fell due. What was due March 1, 1874?

Interest on $3000 for 6 yr. 2 mo., . .	$1110	
" " 180 for 35 yr., ...	378	
Original principal, ...	3000	
	——	$4488

Exercise CXIII.

1. Find the amount due Sept. 10, 1874, on a note of $4500, given July 1, 1871, notes for semi-annual interest having been given.

2. Find the amount due July 1, 1876, on a note dated Jan. 1, 1873, for $3600, notes for semi-annual interest having been given, and no payments having been made.

3. Find the amount of a note drawn for $1700, dated June 10, 1872, notes for semi-annual interest having been given, and no payments made up to the time of settlement, September 21, 1875.

COMPOUND INTEREST.

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359. The **amount** due at the end of each period is regarded as a **new principal** in calculating Compound Interest.

The **Compound Interest** is the difference between the final amount and the original principal.

Compound interest implies *interest* on *interest*. Interest may be compounded annually, semi-annually, quarterly, or for any other period agreed upon. Where rents are collected monthly, the interest may be compounded monthly.

If a sum of money be deposited in a savings-bank, the interest is added to the principal semi-annually. Thus, if the deposit were $10, at the end of 6 mos. there would be placed to the credit of the depositor $10.30; at the end of 6 mos. more, $10.609; at the end of the next 6 mos., $10.93; and at the end of the next 6 mos., $11.26.

The *unit of time* in this case is 6 months.

Let it be required to find the compound interest on $450 at 7% for 2 yr. 6 mos., the interest being added semi-annually.

Solution: Principal, ...	$450.00
Interest for first 6 mos., . . .	15.75
Principal and interest added, .	465.75
Interest on first amount for 6 mos.,	16.30
Second amount, ...	482.05
Int. on second amount for 6 mos., .	17.07
Third amount, ...	499.12
Int. on third amount for 6 mos., .	17.46
Fourth amount, ...	516.59
Int. on fourth amount for 6 mos., .	18.08
Amount at end of 2 yr. 6 mos., .	534.67
Deduct original principal, . .	450.00
Compound int. for 2 yr. 6 mos., .	$84.67

EXERCISE CXIV.

Find the compound interest at 4%, 5%, 6%, 7%, and 8% on

1–5. $450 for 3 yr., interest annually.

6–10. $1200 for 2 yr., interest semi-annually.

11–15. $3564 for 2 yr. 6 mos., interest quarterly.

16–20. $268.40 for 5 yr. 6 mos., interest semi-annually.

21–25. $1400 for 4 yr., interest semi-annually.

NOTE.—Prob. 1, req. comp. int. on $450 at 4%. Prob. 2, at 5%. Prob. 3, at 6%. Prob. 4, at 7%. Prob. 5, at 8%. Prob. 6, int. on $1200 for 2 yr. at 4%, etc.

Calculations can be most easily made by reference to regular tables. These show the several amounts at rates from 2½% to 5%.

As a general rule, compound interest is not allowable. But when interest is due, if the parties so *agree*, it may be added to the principal and draw interest.

A contract is not *void* because it stipulates for the payment of compound interest, but the court will not *enforce* its payment. If payments of interest are not made as they fall due, then simple interest may, by the statutes of some States, be collected on these deferred payments.

PARTIAL PAYMENTS.

360. A partial payment is payment *in part* of a debt—the amount paid and date of payment being endorsed on the back of the note, or other evidence of debt.

The United States Rule.

I. "Apply the payment, in the first place, to the discharge of the interest then due.

II. "If the payment exceeds the interest, the surplus goes toward discharging the principal, and the subsequent interest is to be computed on the balance of the principal remaining due.

III. "If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but the interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal, and the interest is to be computed on the balance as aforesaid."—*Decision of Chancellor Kent of New York, adopted by U. S. Supreme Court.*

361. RULE FOR PARTIAL PAYMENTS IN BRIEF.—*Compute the interest to the date of the first payment, which by itself, or with the addition of a previous payment or payments, exceeds the interest then due. Add the interest to the principal and subtract the payment or payments. The remainder forms a new principal, with which proceed to the next payment in the same manner, until the final settlement.*

EXERCISE CXV.

1. A note for $2000 was given Jan. 4, 1867, on which there were the following payments: Feb. 19, 1868, $400; June 29,

1869, \$1000; Nov. 14, 1869, \$520. How much remained due December 24, 1870, interest at 6%?

Principal, ...	\$2000.00
Interest from Jan. 4 to Feb. 19, '68,.	135.00
First amount, ...	2135.00
First payment, ...	400.00
Balance forming a new principal, .	1735.00
Int. from Feb. 19 to June 29,'69, .	141.69
Second amount, ...	1876.69
Second payment, ...	1000.00
Balance forming new principal, .	876.69
Int. from June 29 to Nov. 14,. .	19.72
Third amount,.	896.41
Third payment, ...	520.00
Balance forming a new principal, .	376.41
Int. from Nov. 14 to Dec. 24, 1870, .	25.09
Balance due on taking up the note, .	\$401.50

2. A note for \$500 was given Feb. 1, 1870, on which were endorsements as follows: May 1, 1870, \$40; Nov. 14, 1870, \$8; April 1, 1871, \$12; May 1, 1872, \$60. What was the balance due on the note, Sept. 16, 1872, interest at 7%?

Principal, ...	\$500.00
Interest to May 1, 3 m., ...	8.75
	508.75
First payment, ...	40.00
	468.75
Interest to May 1, 1872, 2 yr., . .	65.72
	534.37
\$8+\$12+\$60=	80.00
	454.37
Int. from May 1, '72, to Sept. 16, '72,	11.92
Balance due Sept. 16, '72, . . .	466.29

NOTE.—The second and third endorsements, not being singly or together equal to the interest due at the time they were paid, the interest is computed from the first pay-

ment to the fourth, which, taken together with the two previous payments, exceeds the interest due May 1, 1872.

3. A note, bearing interest at 6%, dated April 4, 1872, was given for the payment of $600, on which there were the following endorsements: July 10, 1872, $84.60; Nov. 22, 1872, $10; Apr. 30, 1873, $14; Dec. 5, 1873, $309. What was the balance due on payment of the note, April 5, 1874?

4. $864. BROOKLYN, July 10, 1870.

Four years after date we jointly and severally promise to pay to the order of Thomas Scott eight hundred and sixty-four dollars, with interest. Value received.

SAMUEL POWELL,
SAMUEL BOOTH.

On this note are endorsements as follows: April 6, 1871, $34; June 21, 1872, $300; Feb. 26, 1873, $180; Jan. 1, 1874, $40. What was the balance when the note matured?

EXCHANGE.

362. Exchange means the bartering, or exchanging, of the money of one place for that of another by means of a **Draft** or **Bill of Exchange.**

A Draft is an order by one person on another to pay a third person, or to his order, a certain sum of money at a certain time.

By means of a draft, the third party can, by thus using the credit of the first person with the second, transmit to the distant residence of the second person a large sum without risk.

$1200. CHICAGO, May 22, 1876.

Ten days after sight, pay to the order of John Ross twelve hundred dollars, and charge to the account of

JOHN SMITH.

To Mr. AUGUST BELMONT, New York.

Let us suppose that Mr. Ross wrote on the back of this bill as follows:

"Pay to James Penn, or order.

JOHN ROSS."

and that he then sent the bill to Mr. Penn, in New York, and that Mr. Penn presented it, say on May 26, to Mr. Belmont, who wrote his name across its face, under the word "accepted," and thereby became responsible for its payment.

In the draft, John Smith is the *drawer;* A. Belmont, the *drawee;* and John Ross, the *payee.* Then John Ross became an *endorser;* James Penn, the *endorsee;* and A. Belmont, the *acceptor.*

363. This bill is regarded, in commercial law, as a *foreign* bill. **Domestic bills** *are such as are made payable in the State in which they are drawn. The essentials of a promissory note and of a bill of exchange are the same.*

TO FIND THE COST OF A DRAFT.

RULE.—*Find the percentage on the amount specified at the given rate, and, if it be premium, add it to the face of the draft; if discount, subtract it from the face of the draft.*

EXERCISE CXVI.

1. What will a draft for \$1200 cost, if the premium is 1½%?

2–6. What cost a draft for \$754 at 2½%; \$915.25 at ¾%; \$824 at ½%; \$12634 at 1⅛%; \$9685 at ¼%?

7–10. What cost a draft for \$1500 at 1% discount; \$1260 at 2¼% discount; \$13645 at ⅛% discount; \$96 at 4½% disc't?

The Second and Third are similar to this First bill. In the second of the set, "Second" is inserted instead of "First," and "First and Third unpaid" are written in the parenthesis—meaning, that if the first and third are unpaid, pay *this,* the Second.

Bills drawn so many days after date *need* not be presented before maturity. A sight bill *must* be presented promptly for

acceptance, in order to charge the drawer and endorsers. These are *immediately* liable if acceptance be refused.

364. Purchases made in England must be paid in pounds sterling; French creditors must be paid in francs; German creditors, in reichmarks. So foreigners who owe us must pay us in dollars, and, to do so, must buy of their banker bills of exchange drawn on New York, Boston, etc.

If bankers should charge an excessive price for drafts, persons wishing to pay debts abroad would procure the specie itself and send it. It costs about $1\frac{1}{2}\%$ or 2% to send coin, on account of freight, insurance, and the loss of twenty or thirty days' interest.

A draft on a correspondent in a foreign country is similar in its nature to the bill already described, but differs slightly in form. **A set of three bills** constitutes but **one bill,** as after one has been accepted, the other two are void.

Exchange } NEW YORK, June 1, 1877.
for £2000. } Thirty days after sight of this First of exchange (Second and Third unpaid), pay to the order of Vermilye & Co. one thousand pounds sterling, value received, and charge to account of AUGUST BELMONT.

To BROWN BROTHERS, London.

365. To understand the **Par of Exchange,** as it was stated up to within a very recent period, the following facts must be noted:

In 1792, the eagle contained 247.5 grains of pure gold. Since 1834, the eagle has contained 232.2 grains of pure gold, and the dollar 23.22 grains. Since 1816, the sovereign has contained $113\frac{1}{623}$ grains of pure gold. $113\frac{1}{623} \div 23.22 = 4.8665$. It is plain that the intrinsic value of £1 is \$4.8665. (The alloy is never reckoned in the account.)

The Spanish silver dollar, which was equal to 4s. 6d. old currency, was, and still is, the **basis of exchange.** Up to 1834, £1 was worth but $\$4.44\frac{4}{9}$. The change in the U. S. coinage at that time, introducing lighter gold dollars, was equivalent to a change of $9\frac{1}{2}\%$.

1792, intrinsic par value of £1 =		$4.4444
1834, " "	$9\frac{1}{2}\%$ prem.,	.4221
		$4.8665

Formerly $40 were equal to £9. Under ordinary circumstances bankers now sell sterling at $9\frac{1}{2}\%$ premium on the old par. As, however, when our exports exceed our **imports**, coin must be sent to us, it is possible to buy **drafts** payable abroad at a real discount; that is, **exchange**, instead of being at a premium of $9\frac{1}{2}\%$, is held, or quoted, at 8% or $8\frac{1}{2}\%$, etc.

Required the price of a bill on London for £540 sterling, exchange being at $10\frac{1}{2}\%$ premium.

At the *old par* $40=£9.

$$10\tfrac{1}{2}\% \text{ of } 40 = 4.2. \quad 40 + 4.2 = 44.2.$$

$$\frac{\$44.2}{9} \times 540 = \$2652.$$

When the premium on the old par is given,

To Compute Sterling Exchange.

Rule.—*To* $40 *add the premium, and divide the sum by* 9; *the quotient multiplied by the number of pounds sterling will be the value in American gold dollars. This sum increased by the premium on gold will equal the sum in national paper currency.*

Exercise CXVII.

Required price of the following bills:

1–6. £842 10s. @ $9\frac{1}{2}\%$; £369 12s. 6d. @ 10%; £1265 @ $9\frac{3}{4}\%$; £296 @ $8\frac{3}{4}\%$; £25 8s. 6d. @ 9%; £1600 @ $10\frac{1}{4}\%$.

Since March, 1873, the daily journals have generally quoted, or stated, **the value of the pound sterling directly.** Thus, on Feb. 28, 1877, sterling was quoted at $4.83½@$4.85; the first being for bills drawn at 30 days after sight, the second at sight—that is, as soon as presented. The banker, in the first instance, would have the use of the money about 43 days; in the second instance, for about 13 days. This will appear, if it be supposed that two partners, A. and B., live, the one in

New York, the other in London. Money in the hands of A., in New York, may be made to yield interest every day. The money paid him for a draft drawn on B. need not be paid back until B. accepts it, say from 14 to 44 days afterward. It is plain that the partners get the **interest on the sum in hand,** and can afford to sell exchange at the real par.

7. Required the price of a draft on Liverpool for £1000, exchange being quoted at 4.83½, and the premium on gold at 4½%.

In this case the value of £1 in American coin is directly **stated.**

$$\$4.835 \times 1000 = \$4835 = £1000.$$

As $1.00 in gold costs **$1.045 in currency,** to procure $4835 in gold, there must be paid $\$1.045 \times 4835$ or **$5052.575 in** currency.

366. When the value of £1 is stated at the new par in dollars,

To Compute Sterling Exchange in Currency.

Rule.—*Find the number of gold dollars by multiplying the value of* £1 *expressed in dollars by the number of pounds sterling. Compute the premium on this number of gold dollars, and add it to the number; the result will represent the cost of a draft in currency.*

Find the cost

1. Of a bill of exchange on Liverpool at 3 days' sight for £415 6s., exchange being quoted at $4.88½, gold at 1.04½.

2. Of a bill for £1560 3s. 6d. @ $4.82½, gold at 1.05.

3. Of a bill for £1236 @ $4.84¾, gold at 1.02½.

367*a*. *French Exchange.*

The quotations of French exchange state the **value of a dollar** in francs and centimes. The method of computation is the same in principle as the second method of reckoning sterling exchange. *The number of francs required must be divided by the number of francs in a dollar.*

1. Find the cost of 12000 francs at 5.15 francs to the dollar.

Ans. $2310.10.

Operation: 12000÷5.15=2310.096.

Find the cost in currency

1. Of a bill on Lyons for 1250 francs at 5.17 francs to the dollar, gold being at a premium of $4\frac{1}{2}$%.

2. Of a bill for 11000 francs @ 5.18 gold, premium 3%.

3. The cost in currency of a bill on Paris for 20000 francs @ 5.16, gold being at a premium of $3\frac{1}{2}$%.

Operation: (20000÷5.16)×1.035.

367*b*. *German Exchange.*

The quotations of German exchange state the **value of 4 marks** in American gold. This value is about 95 cents.

Find the cost in currency

1. Of a bill on Berlin for 46460 marks, exchange being quoted at $95\frac{1}{2}$, the gold premium being 4%.

2. Of a bill for 16800 marks at 96, gold premium 4%.

3. Of a bill for 18400 marks at 95, gold premium 5%.

Miscellaneous Problems based on Quotations of February 26, 1877.

Closing Quotations of Foreign Exchange.

	60 days.	3 days.
Prime bankers' sterling bills on London,	$4.83\frac{1}{2}$	4.85
Paris (francs), ...	$5.18\frac{1}{8}$	$5.15\frac{5}{8}$
Antwerp "	$5.18\frac{1}{8}$	$5.15\frac{5}{8}$
Swiss "	$5.18\frac{1}{8}$	$5.15\frac{5}{8}$
Amsterdam (guilders), ...	$.40\frac{1}{4}$	$.40\frac{1}{2}$
Hamburg (reichmarks), ...	$.94\frac{3}{4}$	$.95\frac{1}{4}$
Frankfort "	$.94\frac{3}{4}$	$.95\frac{1}{4}$
Bremen "	$.94\frac{3}{4}$	$.95\frac{1}{4}$
Berlin "	$.94\frac{3}{4}$	$.95\frac{1}{4}$

Find from the foregoing quotations the value in currency of the following, the gold dollar being worth 1.05 in currency:

1. A bill on London @ 60 days for £1700.

2. A bill on London @ 3 days for £444 4s. 6d.
3. A bill on Paris @ 60 days for 2650 francs.
4. A bill on Paris @ 3 days for 13560 francs.
5. A bill on Antwerp @ 60 days for 1200 francs.
6. A bill on Berne @ 3 days for 18000 francs.
7. A bill on Amsterdam @ 3 days for 9600 guilders.
8. A bill on Amsterdam @ 60 days for 840 guilders.
9. A bill on Hamburg @ 60 days for 8000 reichmarks.
10. A bill on Bremen @ 3 days for 18750 reichmarks.

In each of these 10 problems, what would be the *additional* cost if the banker should charge $\frac{1}{8}\%$ as brokerage or commission? (Art. 372.)

Operation: Prob. 1. $\$1.05 \times 1700 \times 4.835 \times .00\frac{1}{8} = \$10.79$.

Income from Stocks and Bonds.

368*a*. Stock Certificates are instruments duly signed by the officers of a corporation stating the number of shares owned by the individual holder, and their par value. These stock certificates, called ***stocks***, are sold, like other merchandise, at varying prices. A purchaser, when stocks are below par, is credited with a larger sum than he actually invests, and *vice versa*. **Stocks are at par** when they will sell for their face. The profits of the corporation are distributed to the stockholders, and vary from 2% to 25%.

Government bonds are really stock certificates, but the dividends do not fluctuate. Different bonds are sometimes named in accordance with their annual rates of interest. Thus, 6% bonds are called 6s.

NOTE.—From ten to twenty lines would hardly suffice to show the distinction between stocks and bonds, a distinction unnecessary in actual computations. It has not been thought best to crowd the text with long definitions. Such as seem desirable are printed in alphabetical order in the APPENDIX. From *Webster's Dictionary* excellent definitions can always be obtained. We have aimed in the text to present only what is absolutely

essential, and thus "*to shun the too much*," believing that it is not wise to needlessly distract the mind of a pupil from the main subject of discussion.

If it were required to find what rate of interest would be realized by investing in U. S. gold-bearing 6s at 120, gold standing at $112\frac{1}{2}$, the operation might be as follows: $6 \times 1.12\frac{1}{2} = 6.75$. $6.75 \div 120 = 5\frac{5}{8}\%$.

Operation: The int. on \$100 would be \$6 in gold, or $1.12\frac{1}{2}$ times \$6 in currency =\$6.75. If \$120 yield \$6.75 int., \$1 would yield \$6.75÷120, or $5\frac{5}{8}$ cents, or $5\frac{5}{8}\%$.

Exercise CXVIII.

1–7. What would be the actual rates of interest received from the following stocks or bonds, viz.: Tennessee 6s bought at 92; Virginia 7s, at 84; Georgia 8s, at 90; New York 6s, at 102; Government 6s (gold at 113) bought at 119; Missouri 7s, at 95; Ohio 6s, at 101?

8. Which will afford the greatest income, Tennessee 6s bought at 82, or Virginia 7s bought at 92?

9. Which will afford the most interest, U. S. 4s at 90, or N. Y. 6s at 105?

10. A capitalist invested \$5000 in each of the following: N. J. State bonds, 7s, at 103; N. Y. 6s at 101; Brooklyn Water Bonds, 6s, at 98; N. C. 6s at 24; and Ga. 7s at 66. What income did he derive from the \$25000 thus invested.

368*b*. To find the Price at which Stocks or Bonds will afford a certain per cent. of income on the Investment.

Rule.—*Divide the income on \$100 by the required rate expressed decimally.*

At what rate must a 6% stock be bought in order to yield 8% of income. *Ans.* \$75.

Operation: 6% of \$100=\$6. \$6÷.08=\$75.

1–5. If an investor desires to secure an income of 8%, at what rate must he buy 5% bonds? At what rate 6% bonds? 7% bonds? 10% bonds? 4% bonds?

368c. It is desirable to be able to determine the amount to be charged as rent in accordance with the value of the property and the percentage required to pay for repairs, taxes, collections, etc., in order that the net result may represent a certain per cent. on the valuation.

To Compute Rentals.

Rule.—*To the net per cent. required add the per cent. required to meet taxes, collections, and repairs, and multiply the value of the premises by the sum of these rates. If either of these items be given as a gross sum, it must be added to the product obtained from the others.*

What rent must be obtained from the lessee of a dwelling-house valued at \$11000, in order to yield the lessor or owner 7%, if the taxes are $1\frac{1}{2}$% on this valuation, the cost of collection of rent is $\frac{1}{10}$%, and if the repairs cost \$200 ?

Operation: $\$11000 \times (.07+.015+.001) = \$946.$
$\$946 + \$200 = \$1146$, or annual rent.

What rent must be obtained from a factory worth \$15000, if the taxes amount to $1\frac{1}{2}$%, the expense of collection of rent to $\frac{1}{10}$%, and the repairs to 2%—the income to net 7% ?

The average annual rental of a certain house was \$982.35. Of this \$100 per annum was spent in repairs, $1\frac{9}{10}$% of the remainder was required for taxes, and $\frac{1}{10}$% for incidentals. The owner sold the house for \$20000, and invested this sum in U. S. 6s at 112. If the gold coupons, or interest certificates, sell at 104, what addition to his income has the person made by this transaction ? *Ans.* \$428.41.

Profit and Loss.

369. The **terms** used in Profit and Loss, the **signs** of these

terms, and the **general terms of percentage with which they correspond** are as follows :

Cost,	C.	Base.
Gain or loss per cent.,	g.% or l.%.	%
Gain or loss,	g. or l.	Percentage.
Selling Price,	S. P.	Amount or difference.

The g.% or l.% is always a **decimal**

Rules.

The g. or l.÷C.=g.% or l.%.

C.+g., or, C.—l.,=S. P.

$$\frac{\text{S. P.}}{1+\text{g.\%}}, \text{ or, } \frac{\text{S. P.}}{1-\text{l.\%}}=\text{C.}$$

Exercise CXIX.

369*a*. Required the **g.%** or **l.%** in the following **instances** :

1. A comb bought for 16 cts. was sold for 20 cts.

2. Cloth was bought for 80 cts. per yd. and was sold for 75 cts.

3. 980 lbs. almonds, bought for $153, were sold for $184.

4. A sleigh cost $40 and was sold for $45.

5. Hay bought at $9 was sold at $10.50.

370. Required the **S. P.** of the following :

6–10. Bought muslin at 12 cts. a yd., and sold different lots at a gain of 10%; 12½% ; 15% ; 25% ; 30% ?

11–15. A man bought 5 boat-loads of lumber, each at $4368. He sold the first at a gain of 12% ; on the second, of 45% ; on the third, a loss of 12% ; on the fourth, of 42% ; on the fifth, of 18%. What did he receive in all?

16. A man bought 90 gallons of syrup, at $1.20 a gallon. Ten gallons were lost by leakage. He sold the remainder, and gained 12½% on the whole cost. At what price per gallon ?

371. Required the **cost** :

17. S. P.=$4.05, g.% 12½.	20. S. P. $55, g.%=10.
18. S. P.=$125, l.%=15.	21. S. P. $55, l.%=25.
19. S. P. $67.50, l.%=10.	22. S. P. .06¼, g.% 400.

MISCELLANEOUS.

23, 24. A miller sold corn at \$1 a bushel, and gained 25% ; he then sold a lot for \$54, on which he gained 30%. For what did he sell this per bushel, and how many bushels did he sell ?

25. Bought a horse for \$130, on a credit of 9 months, but sold him immediately for \$120 cash. If the use of the money were reckoned at 7%, what was lost ?

26. Bought flour, and sold a part at \$9 a barrel, gaining 25% ; was obliged to sell the remainder at \$6.48 a barrel. What was the gain % or loss % on this ?

27. A bbl. of turpentine was bought for \$10. A part having leaked out, the remainder was sold at 55c. a gallon, and a gain of 15% was realized on the whole. How many gallons were lost ?

28. If when the S. P. is \$75 the g.% is 15, what would the l.% be if the S. P. were \$40 ?

29. If $\frac{4}{5}$ of a quantity of sugar was sold for what the whole cost, what was the g.% on what was sold ?

30. What would be the gain % on the part sold if the cost of an article were realized from the sale of $\frac{7}{8}$ of it ?

31. What would be the gain % if $\frac{3}{4}$ of a barrel of flour were sold for as much as the whole barrel cost ?

Commission and Brokerage.

372. A person who transacts business for another performs a commission. The *commercial* definition of **commission is the compensation allowed to brokers or commission merchants** when they act as agents for others.

373. Brokerage on stocks and money is usually computed on the **par value** *as a base*, but the brokerage or commission for the purchase or sale of *other merchandise* is reckoned on the **amount received or expended** *as a base.*

The following abbreviated RULES will serve for practical purposes:

I. Base $\times$ r = Commission or Brokerage.

II. The investment *plus* the commission = Total amount.

III. Total amount $\div (1 + r\%)$ = Investment.

NOTE.—The *total amount* includes both the sum to be invested by the broker or buyer and the broker's commission.

EXERCISE CXX.

Find the brokerage for selling each of the following lots of stock at their par value at every one of the specified rates of brokerage; viz.:

1–36. 5000 @ 1%?	1500 @ $1\frac{1}{2}$%?
7500 @ $\frac{1}{2}$%?	9400 @ $2\frac{1}{2}$%?
4200 @ $\frac{1}{4}$%?	10200 @ $\frac{1}{8}$%?

1. Sent a broker $10756.50, with orders to invest it in N. C. 7s, at 75. The broker retained a commission of $\frac{3}{4}$% on the par value of the stock out of the amount sent him. How many $100 bonds did he buy?

Explanation.—$\frac{3}{4}$% on par is 1% on 75; hence the broker invested $100 out of every $101 sent him, or, in all, $10650. At 75, he could purchase 142 bonds.

2. What draft must a broker draw on his principal, or employer, to cover his commission, 2%, and the cost of a bill of goods invoiced at $2250?

3. If a man pay $45 for the collection of $2400, what was the rate per cent. paid?

4–13. A lawyer charging 20% for collection, received from one client $900; from another, $75; from another, $84.50; from another, $172; from another, $18. What were the respective amounts collected? What would the several amounts have been if he had charged 5%?

14. What amount must be sent to an agent to buy 450 bbls. of beef at $16.50, and pay storage at 5c. a bbl., insurance at 60c. a $100, and brokerage at $\frac{3}{4}$% on money thus employed?

15. Sent $9115.86 to a broker in Chicago, with directions to purchase beef at $14.50 per bbl., to insure it for 30 days at 12c. a $100, to pay storage at 4c. a bbl. for 10 days, the broker to receive 1% commission on money thus expended. How many bbls. of beef did he buy? (Find sum actually expended for 1 bbl., commission included.)

16. Sent a broker $6544.80 to invest in Lake Shore R. R. Stock @ 54. How many $100 shares did he buy, he retaining his commission of 1% on the amount invested out of the amount received? *Ans.* 120 shares.

Insurance.

374. The contract of Insurance is called a **Policy.** It specifies the *property* insured, the *amount* to be paid in case of loss, the *rate* charged, the *parties* concerned, and the cautionary *conditions.*

375. The sum paid for the policy is called the **Premium.** It is computed at a certain **Rate** on the sum assured.

376. To find the Premium.

Rule.—*Multiply the sum assured by the rate.*

Exercise CXXI.

What premium must be paid to cover the following risks at each of the specified rates? viz.:

1–6.	$4500 @ 1%?	19–24.	$1260 @ $2\frac{1}{2}$%?
7–12.	$4800 @ $\frac{1}{2}$%?	25–30.	$14890 @ $1\frac{3}{4}$%?
13–18.	$10640 @ $\frac{1}{4}$%?	31–36.	$9000 @ $\frac{1}{8}$%?

377. To find the Face of a Policy covering both the Property and the Premiums.

Rule.—*Divide the amount insured by* 1 *less the rate.*

Suppose that it were required to insure a house worth $4000 for its full value, the rate to be paid being 1%, and to provide that in case of its destruction the owner should recover the

value and also the premium paid for the assurance, the problem might be solved thus: $4000 \div (1-.01)$ or $.99 = \$4040.40$.

Exercise CXXII.

Required to find the face of the policies which will yield in case of loss the following values, plus the premiums paid at each of the specified rates; viz.:

1–6. \$8000 @ $\frac{1}{2}$%.	19–24. \$1500 @ $\frac{1}{3}$%.
7–12. \$12460 @ 4%.	25–30. \$4500 @ $\frac{45}{100}$%.
13–18. \$15200 @ $2\frac{1}{4}$%.	31–36. \$3750 @ $\frac{3}{8}$%.

Taxes.

378. The money required by Government is obtained by **taxation.** The General Government secures what it requires by **indirect taxation** in the form of **duties on imports,** from **excise duties** obtained from the sale of revenue stamps, and from the sale of postal stamps, etc. Money required by the State or city is raised chiefly by **direct taxation** on property. **Real Property,** or real estate, consists in houses and lands. **Personal Property** consists in movables, such as furniture, goods, money, etc. Real estate is usually assessed at from $\frac{1}{10}$ to $\frac{1}{2}$ of its actual value. A **Poll Tax** is a small sum levied, in some communities, on each adult male citizen, whether rich or poor.

The gross amount necessary is determined by the Legislature. To each county is assigned a certain amount, based on the county valuation. This tax will represent a certain per cent. of the total county valuation as fixed by the assessors. Each individual will be compelled to pay a tax proportioned to his property, real and personal.

The tax set down as the share of a certain town is \$4000. The total valuation is \$100,000. The adult males liable to a poll tax number 100; these are each taxed \$1. The town collector is allowed 5% for collecting the tax. With these

conditions it is required to find the rate of taxation; and, also, to determine what A. shall pay, whose property is assessed at $1500, and who has two adult sons?

OPERATION.

$$\$4000 \div .95 = \$4210.52 \text{ (this includes collector's fee).}$$
$$\$4210.52 - \$100 = \$4110.52 \div \$100000 = .04\tfrac{11}{100}, \text{ or the rate \%.}$$
$$(\$1500 \times .0411) + \$3 = \$64.65.$$

379. TO ASSESS A TOWN OR COUNTY.

I. *To the amount assigned by the State add the cost of collection, which is found by dividing the amount by a unit diminished by the rate allowed the collector.*

II. *From the whole tax subtract the amount of the poll tax.*

III. *Divide the remainder, or property tax, by the amount of taxable property; the quotient, expressed in hundredths, will represent the rate of taxation.*

IV. *Multiply the assessed value of each man's property by the rate, and to the product add his poll tax.*

If a tax of $508000 be levied on a county assessed at $25,600,000, what is the rate, provided the cost of collection is 1%? What must a man pay whose property is assessed at $4500? *Answers:* The rate is $2\frac{5}{1000}$%; the tax, $90.23.

If a tax of $5097 be levied on a town, the taxable property of which is assessed at $849500, what is the rate, and what the tax on property assessed at $3570? *Ans.* $21.42.

The taxable property of the city of X., for a certain year, was $50,000,000, the tax declared was 2½ cents on a dollar. What was the amount to be raised?

Mr. Brown paid tax on $15000; Mr. Jay, on $4500; Mr. Hay, on $12250; Mr. Dana, on $6400; Mr. Fox, on $14250; and Mr. Troy, on $3500, at 2½%. What did each pay the city collector?

380. Duties are usually paid through the agency of a **custom-house broker,** in order to avoid errors and to save time.

381. Allowances are sometimes made which go to reduce the duty, such as **Draft,** which is an allowance for waste, and is to be deducted first; **Tare,** for weight of box or bale; **Leakage**, for waste of liquids; **Breakage**, for bottles, etc.

Gross weight is the amount before allowances; *net weight* is the weight after all allowances have been deducted.

382. There are frequent changes made in the regulations or laws for the collection of duties and in the rates charged. The actual performance of the work demands much local knowledge and not a little skill in matters of exchange, etc. The limits of a school arithmetic will not afford space for practical directions. The general theory will be obvious to any pupil familiar with Analysis and Percentage.

EQUATION OF PAYMENTS.

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383. ***Equation of Payments*** is the process of finding the **time** when the **sum** of several debts due at different dates **should be paid.**

Mr. Blaine owes Mr. Tilden $1500, of which $500 will be due in 3 mos., $500 in 6 mos., and $500 in 12 mos. At what time may B. equitably pay the whole sum?

$$\begin{array}{rr} 500\times 3 = & 1500 \\ 500\times 6 = & 3000 \\ 500\times 12 = & 6000 \\ \hline 1500) & 10500(7. \end{array}$$

Explanation: On the 1st item B. is entitled to the use of $1500 for 1 mo.; on the 2d item, to $3000 for 1 mo.; on the 3d item, to $6000 for 1 mo.; or, in all, to the use of $10500 for 1 mo., which is equivalent to the use of $1500 for 7 months.

RULE.—*Multiply each debt by its term of credit; then divide the sum of the products by the sum of the debts.*

EXERCISE CXXIII.

1. A man owes \$1000. Of this \$200 is now due, \$200 will be due in 3 mos., \$400 in $4\frac{1}{2}$ mos., and the remainder in $6\frac{1}{2}$ mos. What time after the present should a note for \$1000 be dated in order to liquidate the whole?

2. If B. owes a man \$150, payable in 2 mos., \$350 in 4 mos., and \$760 in 9 mos., at what time may he be called on to pay a note drawn for the sum of the debts?

3. The total of the several purchases made by a merchant is \$4000. For one invoice of \$1200 he has given his note for 4 mos.; for another of \$800, a note at 6; for another of \$1500, he has agreed to pay cash; and for \$500, he is to have 1 year's credit. How much time should he be allowed on a note made for \$4000 in payment of the whole?

4. A. owes \$800, payable in 9 mos. At the end of 3 mos. he pays \$400. Three months later he pays \$200. How long beyond the 9 mos. may he defer the payment of the amount still due?

5. Sold merchandise to Mr. Clark as follows:

May	10,	\$40.00 ×	0 =	0
"	15,	65.38 ×	5 =	325
"	27,	90.50 ×	17 =	1,530
"	30,	120.40 ×	20 =	2,400
June	6,	50.20 ×	26 =	1,300
"	20,	110.90 ×	40 =	4,400
"	25,	148.00 ×	45 =	6,660
		\$625.38	623)16,615(27 days aft. May 10.	

When the bill is paid, the amount will equal \$625.38, plus interest from June 6 (27 days after May 10).

NOTE.—The cents having been omitted in computing the products, they should be omitted in the divisor.

6. Sold merchandise to Mr. Clark as follows:

July 1,	bill @	3	mos.	credit,	$400 × 94 =	$37,600
" 5,	"	4	"	"	500 × 128 =	64,000
" 10,	"	4	"	"	500 × 133 =	66,500
" 20,	"	6	"	"	1500 × 203 =	304,500
Aug. 10,	"	3	"	"	200 × 133 =	26,600
" 20,	"	2	"	"	100 × 113 =	11,300
Sept. 15,	"	3	"	"	250 × 168 =	42,000
					$3450	)552,500
						160

This total will be due 160 days after July 1.

Mr. Clark, the debtor, may be considered as a creditor for 552500 days' interest on $1, counting from July 1.

NOTE.—In some States, by *statute*, a dealer may charge interest on a balance due, after a statement has been submitted to the debtor, and its correctness has been admitted. By *common law*, running accounts draw no interest. The party who has given the most credit must submit statements of balances to the other party and secure his acknowledgment of their correctness, in order that he may secure the difference in interest in his favor.

Averaging Accounts.

384. The *balance* of an account is the difference between the sum of the debits and the sum of the credits.

The interest balance is the difference between the total interest on the debit items, and the total interest on the credit items.

385. The cash balance is a result obtained by adding to the *Dr.* side the interest on each debit item up to the date of settlement, and adding to the *Cr.* side the interest on each credit item up to the same date, and then finding the difference between two amounts.

1. Let it be required to find when the **balance** of the following ledger account falls due.

Dr. JOHN WENTWORTH. **Cr.**

1876.						1876.					
May	1	To Mdse.,	$	350	50	May	10	By Mdse.,	$	200	00
"	8	" "		50	00	"	20	" "		190	00
"	15	" "		400	00	"	30	" Cash,		340	50
"	30	" "		100	00	June	1	" "		28	30
June	10	" "		175	75	"	12	" Mdse.,		296	70
July	31	" "		56	56	Aug.	15	" "		41	70
Aug.	10	" "		200	00	Nov.	10	" Cash,		300	00
Sept.	10	" "		300	00			Balance,		235	61
			$	1632	81				$	1632	81
		To Balance,		235	61						

Rule.—*Consider the earliest date as a "focal" date, and reckon the days from it up to the time when each account becomes due. Multiply each item on both sides by its corresponding number of days. Find the sum of the products on each side, and divide their difference by the difference between the debits and credits. The quotient represents days or time. This must be added to the focal date when the balances are on the same side, and must be subtracted from the focal date when the balances appear on opposite sides.*

May 1, $351 × 0 = 0
" 8, 50 × 7 = 350
" 15, 400 × 14 = 5600
" 30, 100 × 29 = 2900
June 10, 176 × 40 = 7040
July 31, 57 × 91 = 5187
Aug. 10, 200 × 101 = 20200
Sept. 10, 300 × 132 = 39600
$1634 80877
1398
236)10116(43 days.
944
676
708

May 10, $200 × 9 = 1800
" 20, 190 × 19 = 3610
" 30, 341 × 29 = 9889
June 1, 28 × 31 = 868
" 12, 297 × 42 = 12474
Aug. 15, 42 × 106 = 4452
Nov. 10, 300 × 193 = 57900
$1398 90993
80877
10116

The balances are on opposite sides in this account.

May 1, 1876—43 d.=Mar. 19, 1876. Mr. Wentworth should date his note, drawn for $235.61, Mar. 19, 1876.

386. In thus estimating the time from the earliest date, students should comprehend that each product represents the interest on $1 for the number of intervening days. Mr. W. is entitled to credit for the products on the Dr. side, and should pay interest for the products on the Cr. side, as these last *are in favor of Mr. W.'s debtor.*

Mr. W. then being credited with 80877 days' interest on $1, and being debited with 90993 days' interest on $1, must in some way charge himself with the difference. This he may do by so dating the note which he will give for the balance as will give the payee 10116 days' interest on $1. The interest, $235.61 for 43 days, equals this. May 1 was assumed as the time when every item fell due, therefore this note must be dated Mar. 19, 1876.

Exercise CXXIV.

When is the balance of the following account due?

Dr. HEISTER CLYMER. *Cr.*

1875.				1875.			
Jan.	15	Mdse. at 6 mos.,	$500	Mar.	10	Cash, ...	$200
Feb.	20	" " 4 "	300	April	30	" . . .	300
Mar.	30	" " 6 "	400	June	1	Note at 2 mos.,	500
Apr.	20	" " 3 "	300	(Amount due June 5, $1000.)			
(Amount due Sept. 2, $1500.)				(Bal. due Feb. 27, '75, $500.)			

387. Average each account separately (see *Equation of Payments*). Then multiply the sum of the smaller side by the number of days intervening between its date and the date of the larger side, and divide by the balance of the account, and the result will, *in this case*, be the number of days in which the balance will fall due *after* the date of the larger side. Why?

Accounts Current with Cash Balance.

388. An account current is a statement to a customer of his debits and credits, with the date at which each item was due, with the interest on each up to the time of settlement, if the **Cash Balance** be included.

EXAMPLE.

Dr. JOHN PALMER *in account with* J. SMITH. *Cr.*

Date.		Merchandise.	Days.	Int. 6 per c.		Date.		Merchandise.	Days.	Int. 6 per ct.	
May	1	$145.50	111	$2	69	Apr.	20	$400.00	122	$8	13
"	10	298.50	102	5	07	May	12	96.50	100	16	09
"	31	741.00	81	12	47	June	4	38.90	77		49
June	1	296.80	80	4	95	July	10	520.30	41	3	55
July	20	344.30	31	1	77						
						Aug.	20	1055.70		28	26
Aug.	20	1826.70		26	95	Int.		28.26			
Int.		26.95									
								1093.96			
						Bal.		759.09			
		$1853.05						1853.05			

Aug. 20. To Bal., $759.09.

RULE.—*Find the interest at 6% on each item up to the time of settlement, and add the sum on each side to the sum of the account on the same side. The difference of the amounts will be the cash balance.*

EXERCISE CXXV.

Dr. D. APPLETON & CO. *in acc't with* ROE LOCKWOOD. *Cr.*

1876.					1876.				
Mar.	10	To Mdse.,	$540	20	Mar.	1	By Mdse.,	$200	40
May	1	" "	760	80	Apr.	1	" Cash,	360	84
June	10	" "	1200	00	May	4	" Mdse.,	1000	00
July	10	" "	460	90	June	7	" Cash,	400	00
Aug.	6	" "	325	25	July	10	" Mdse.,	1000	00
Sept.	1	" "	474	62	Sept.	4	" Cash,	100	00
Oct.	4	" "	917	18	Nov.	9	" Mdse.,	2000	00

Required cash balance Dec. 1, 1876.

Dr. John Ross *in acc't with* D. Field. *Cr.*

1876.					1876.				
Jan.	10	To Mdse.,	$600	00	Jan.	4	By cash,	$500	00
"	15	" "	500	00	"	9	" "	400	00
Feb.	4	" "	400	00	Feb.	3	" "	600	00
Mar.	7	" "	600	00	Mar.	4	" "	400	00
Apr.	4	" "	800	00	Apr.	5	" "	700	00
May	10	" "	300	00	May	10	" "	400	00
June	7	" "	842	00	June	9	" "	400	00

Required cash balance, July 1, 1876.

RATIO.

389. *Ratio* is the relation which exists between two numbers. It equals the quotient found by dividing the one by the other, the divisor being the standard or measure. The ratio of 12 to 4 equals 3. It is expressed thus: $12:4=3$. The **colon** may be regarded as a **sign of division.** The number first named is called the **antecedent,** the second number is called the **consequent.** The two are called the **terms** of the ratio.

390. The two terms of a ratio have the same relation to each other as the terms of a fraction. Thus, the expression $12:4$ is equivalent to the fractional expression $\frac{12}{4}$. It is read thus: "The ratio 12 to 4."

Oral Exercise.

What is the ratio of 4 to 2; of 6 to 3; of 12 to 3; of 16 to 4; of 8 to 8; of 4 to 8; of 25 to 5; of 5 to 25; of $\frac{3}{4}$ to $\frac{1}{4}$; of $\frac{1}{4}$ to $\frac{3}{4}$; of $\frac{5}{8}$ to $\frac{3}{8}$; of $\frac{3}{8}$ to $\frac{5}{8}$; of $\frac{4}{5}$ to $\frac{2}{5}$; of $\frac{6}{9}$ to $\frac{3}{9}$; of $\frac{6}{7}$ to $\frac{3}{7}$; of $10 to $5; of 12 pecks to 4 pecks; of 6 bu. to 6 pecks; of 6 pecks to 24 pecks; of $\frac{30}{7}$ to $\frac{3}{7}$?

Which is the greater ratio, 15 : 2, or 16 : 3? Why?

Which is the greater ratio, 21 : 7, or 7 : 21? 16 to 15, or 15 to 16? 9 : 3, or 12 : 4?

Which is the greater, $\frac{9}{3}$ or $\frac{12}{4}$? $\frac{25}{5}$ or $\frac{10}{2}$? $\frac{15}{2}$ or $\frac{16}{3}$?

What ratios are equal to 2? What to 3?

As multiplying the numerator of a fraction multiplies its value, so multiplying the antecedent multiplies the ratio; and in general the principles of fractions will apply to ratios.

Proportion.

391. A proportion expresses the equality of two simple ratios. Thus, $4 : 2 = 6 : 3$, or, as usually written, $4 : 2 :: 6 : 3$. This proportion is read, Four is to two as six is to three, meaning that the ratio 4 to 2 equals the ratio 6 to 3.

The first pair of the four terms of a proportion is called the **first couplet,** the second pair, the **second couplet.** The first and fourth terms are called the **extremes,** the second and third are called the **means** of the proportion.

392. As the consequent is a divisor and the ratio the corresponding quotient, **the antecedent is the product of the consequent and ratio.** (See *Principles of Division.*) It follows that, instead of the antecedent of a couplet, we may write the product of the consequent and ratio. Thus, as $10 : 5 :: 4 : 2$, so $5\times2 : 5 :: 2\times2 : 2$; and, if $a : b :: c : d$, so $b\times r : b :: d\times r : d$.

Now, as $5\times2\times2=5\times2\times2$, and as $b\times r\times d=b\times d\times r$, it appears that the product of the extremes of a proportion is equal to the product of the means.

Principles.

I. *The ratios of the couplets of a proportion are equal.*

II. *The product of the extremes of a proportion equals the product of the means.*

III. *If four numbers when placed in order are such that*

the products of the extremes and means are equal, they are proportionals.

From Principles I. and II. it follows that the terms of a proportion may be arranged in at least eight different ways, and the factors of each pair remain wedded:

6 : 3 : : 10 : 5	10 : 5 : : 6 : 3
6 : 10 : : 3 : 5	10 : 6 : : 5 : 3
3 : 6 : : 5 : 10	5 : 10 : : 3 : 6
3 : 5 : : 6 : 10	5 : 3 : : 10 : 6

Application of Principle I.

393. Pupils versed in fractions should be able, when three terms of two equal fractions are given, to readily ascertain the fourth term. Thus, in these four questions, marked *a, b, c,* and *d,* the ratio expressed by the complete fraction must divide the odd numerator, or multiply the odd denominator.

$$a.\ \frac{6}{3}=\frac{10}{(\ )};\ b.\ \frac{6}{3}=\frac{(\ )}{5};\ c.\ \frac{6}{(\ )}=\frac{10}{5};\ d.\ \frac{(\ )}{3}=\frac{10}{5}.$$

The same operation may be performed on the numbers arranged as a proportion; for the two couplets are as two fractions. If three out of four of the terms of a proportion be written, at least *one couplet must be complete.* From this couplet we can determine the common ratio of both couplets. We may apply it to the odd term, using it as a *multiplier* with the *second* term of the incomplete couplet, or as a *divisor* of the *first* term of the incomplete couplet. Thus, in the proportion:

9 : 3 : : 12 : ( ), $12\div3=4$, the 4th term.
10 : 2 : : ( ) : 7, $7\times5=35$, or the 3d term.
7 : ( ) : : 42 : 6, $7\div7=1$, the 2d term.
( ) : 15 : : 4 : 8, $15\times\frac{1}{2}=7\frac{1}{2}$, the 1st term.

Application of Principle II.

394. In applying Principle II. we may reason thus: If three terms of a proportion be written, two must either be extremes

or they must be means. The product of this pair divided by the odd quantity must give the mate of the odd one. In the problem $9:3::12:d$, $d=\frac{3\times12}{9}$.

In the problem $10:2::c:7$, $c=\frac{10\times7}{2}$.

In the problem $7:b::4:26$, $b=\frac{7\times6}{42}$.

In the problem $a:15::4:8$, $a=\frac{15\times4}{8}$.

Simple Proportion.

395. The method of finding the fourth term of a proportion when three terms are given, is often called the **Rule of Three.** So numerous and valuable are its applications that it is sometimes called the GOLDEN RULE.

Let us form a proportion which has reference, not to abstract numbers, but to actual things.

6 yds. : 3 yds. : : \$10 : \$5.

Restated partially (see preceding Article), we should have four questions:

1st. ( ) : 3 yds. : : \$10 : \$5 ?
2d. 6 yds. : ( ) : : \$10 : \$5 ?
3d. 6 yds. : 3 yds. : : ( ) : \$5 ?
4th. 6 yds. : 3 yds. : : \$10 : ( ) ? or,

1st. How many yards of cloth can be bought for \$10, if 3 yards cost \$5? etc., etc.

As a multiplier or divisor is necessarily abstract, so the ratio is always abstract. The result takes the name of the multiplicand or of the dividend—the ratio being a multiplier if the 1st or 3d term be required, and a divisor if the 2d or 4th term be called for.

The four questions indicated above are solved as follows:

1st. 3 yds. $\times\frac{10}{15}$ or 6 yds. 3d. \$5 $\times\frac{6}{3}$ or \$10.
2d. 6 yds. $\div\frac{10}{5}$ or 3 yds. 4th. \$10 $\div\frac{6}{3}$ or \$5.

In practice, the **required term** is generally made to fall in the *fourth place.*

Arranging the terms of a problem is called "**Stating the question.**"

396. To State a Question in Proportion.

Rule.—*Write for the third term that quantity which is like the answer sought.*

If the answer must be greater than this third term, write the greater of the two remaining quantities in the second place; if less, then write the smaller of the two in the second place; the other quantity must be written as a first term.

A. *Divide the product of the second and third term by the first term. Or*

B. *Divide the third term by the ratio of the first and second.*

Exercise CXXVI.

397. Where the value, weight, dimensions, materials, or force depends on the number of things given, no mistake is likely to occur in stating the question.

1. If three caps cost $5, what cost 12 caps?
2. If 4 bricks weigh 15 lbs., what will 2 bricks weigh?
3. If 5 coats require 20 yds., what will 15 coats require?
4. If 7 bu. of coal feed an engine 2 hours, how many hours will 35 bu. feed it?

In these questions, as *more requires more* or *less requires less*, the **ratio** is **direct.**

Exercise CXXVII.

398. Where *more requires less* or *less requires more,* as in questions such as follow, pupils sometimes fail to discriminate, and so fall into error.

1. If 2 men can make a certain number of boxes in 3 days, how long will it take 8 men to do as much? *Ans.* $\frac{3}{4}$ of a day.

2. Boron weighs twice as much as water; platinum weighs 22 times as much as water; if 2500 lbs. of boron occupy 20 cubic feet, how much space will 2500 lbs. of platinum occupy?

Ans. $1\frac{9}{11}$ cu. ft.

3. If 3 horses consume 40 bushels of oats in 28 days, how long will it require 21 horses to eat as much? *Ans.* 4 days.

In such questions, the ratios are **inverse.**

399. *Questions having direct Ratios.*

Exercise CXXVIII.

1–8. If 3 cu. ft. of cork weigh 45 lbs., what is the weight of 10 cu. ft.? 16 cu. ft.? 27? 12? 112? 230? 709? 4004?

9–12. If 2 cu. ft. of water weigh 125 lbs., what will 12 cu. ft. of water weigh? 14 cu. ft.? 27 cu. ft.? 42 cu. ft.?

Make questions, similar to these just stated, respecting each of the substances described as follows:

One cubic foot of sheet brass weighs about 513 lbs.; of copper, 543; of zinc, 450; of cast-iron, 450; of wrought-iron, 486; rolled lead, 712; of mercury, 849; of silver, 657; of tin, 456; of gold, 22 carat, 1093; of ash-wood, 52.8; of cedar, 35; chestnut, 38; hickory, 49; lignum-vitæ, 83; Eng. oak, 58; live-oak, 67; white pine, 35; spruce and bl. walnut, 31; coal, anthracite, 96; coal, cannel, 96; coal, bituminous, 80; coke, 62.5; cotton, pressed, 20; clay, 120; dry sand, 120; granite, 166; hay, 25; limestone, 198; marble, 168. (Haswell's Tables.)

Thus: If the zinc required to fill a certain space weigh 900 lbs., what would an equal bulk of wrought-iron weigh?

Statement: 450 : 486 : : 900 : (?)

If a cu. ft. of water weighs 1000 oz., what will a cu. ft. of 22-carat gold weigh?

Statement: 62.5 : 1093 : : 1000 : (?)

The nearest travelling distance from New York to Albany

is 150 miles; to Buffalo, 433 miles; Cleveland, 594; Chicago, 898; St. Louis, 1112; New Orleans, 1516; Cincinnati, 772; Pittsburgh, 431; Philadelphia, 88; Baltimore, 186; Washington, 226; Richmond, 356; Charleston, 813; Savannah, 917; and to Montgomery, 1120 miles. (Haswell.)

If it cost $9.00 for R.R. fares to Buffalo, what would the fare to Albany be at the same rate? What the fare to Cleveland? To Cincinnati? Chicago? Richmond? Etc.

Exercise CXXIX.

1. A wine gallon contains 231 cu. in., a beer gallon 282 cu. in., how many wine-gallon measures will fill 12 beer-gallon measures?

2, 3. If 2 lbs. of beef cost 44 cts., what will 11 lbs. cost? 16 lbs.?

4, 5. If 3 bbls. of flour cost $25, what will 14 bbls. of flour cost? 17 bbls.?

6. If 4 men can mow 6 acres of grass in a day, how many acres can 11 men mow in the same time?

7, 8. When eggs are sold at 12 for 20 cts., what will 18 eggs sell for? 32 eggs?

9–11. If the shadow of a 10-ft. pole extends 7 ft., how long is the shadow of a tree which is 60 ft. high; 85 ft. high; $210\frac{1}{2}$ ft. high?

12. A. and B. share $1800, A. taking $2 as often as B. takes $7; what will each get? ("Distributive Proportion.")

13–15. If 9 cows are worth $685, what is the value of 13 cows? Of 17? Of 28? Of 3? Of 11? Of 2116?

16–18. If 4 yards of cloth cost $17, how many yards can be bought for $102? How many for $119? For $153?

19–22. If $\frac{3}{5}$ of a barrel of sugar cost $19, what will $\frac{3}{7}$ of a barrel cost? $\frac{3}{12}$ of a barrel? $\frac{2}{11}$? $\frac{9}{17}$?

23. Mr. S. owes A. $1650, B. $2140, and C. $3164. He has failed and can pay but 60 cts. on each dollar of his indebtedness; what will each receive?

24. When $3200 gold can be bought for $3582.50 currency, what sum in gold will $5000 currency purchase?

25. If $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{8}{9}$ of a bbl. of flour cost £$\frac{7}{8}$, how many bbls. can be bought for $2\frac{4}{5}$ guineas.

Stated: £$\frac{7}{8}$: $2\frac{4}{5}$ gui. : : $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{8}{9}$: (?)

$$\textit{Performed:}\ \frac{\cancel{8}}{\cancel{7}} \times \frac{\overset{3}{\cancel{21}}}{\underset{\underset{5}{\cancel{10}}}{\cancel{20}}} \times \frac{\overset{\cancel{2}}{\cancel{14}}}{5} \times \frac{3}{\cancel{4}} \times \frac{\overset{3}{\cancel{6}}}{\cancel{7}} \times \frac{\overset{2}{\cancel{8}}}{\cancel{9}} = \frac{54}{25} = £2\frac{4}{25}.$$

26–28. If sound traverses water at the rate of 4708 feet in a second, how long after a submarine explosion might it be heard by a listener 40 miles distant? 60 miles? 79 miles?

29. If the flash was seen 13 seconds before the report of a cannon, how far was the gun away, if sound travels through air 4372 feet in 4 seconds?

30. A man who had purchased a barrel of spirits for $31.50 added a quantity of water to it, and then sold the fluid at 40 cents a gallon, and made a profit of 10 cents on each gallon bought. How much water did he add?

31–42. A difference of 15° of longitude makes a difference of 1 hour in clock-time. How far E. or W. of New York is each of the following places, the difference in time for Funchal being $3\frac{48}{60}$ hrs.; that of Cadiz, $4\frac{31}{60}$ hrs.; Genoa, $5\frac{33}{60}$; Constantinople, $6\frac{52}{60}$; Bombay, $9\frac{48}{60}$; Calcutta, $10\frac{49}{60}$; Canton, $12\frac{29}{60}$; Jeddo, $14\frac{16}{60}$; Cape of Good Hope, $6\frac{10}{60}$; St. Helena, $4\frac{33}{60}$; St. Thomas, $\frac{36}{60}$ (slow); New Orleans, $1\frac{4}{60}$ (slow)?

NOTE.—The computation of interest may be performed by proportion; in fact, a thorough explanation of problems in interest almost necessitates the use of the principles of proportion. Take the following illustration:

Required the interest on $124 at 7%, for 2 years and 2 months. *Ans.* $18.81.

(*a.*) $1 : $124 : : 7 cents : $8.68.
(*b.*) 1 year : $2\frac{1}{6}$ yrs. : : $8.68 : $18.81.

400. *Questions having Inverse Ratios.*

When the quantities are so related that as the one increases the other diminishes, or as one is decreased the other must be increased, they vary inversely and their ratio is inverse.

Exercise CXXX.

1. If a room requires 84 yards of carpeting $\frac{5}{8}$ of a yard wide, how many yards of carpet $\frac{3}{4}$ of a yard wide would it require?

2. In what time will 42 sheep consume as much as will feed 36 sheep 29 days?

3. If loaves weigh 18 oz. each when flour is $8 a barrel, what ought loaves to weigh when flour is $10.50?

4. When $113 currency would purchase $100 gold, 800 gold dollars were bought; what number of gold dollars will the same amount of currency buy when gold is quoted at $1.15\frac{1}{2}$?

5. How long will it take 18 men to do as much work as 15 men can do in 25 days?

6. If a package weighing 100 lbs. can be carried 42 miles for a certain sum, how far may 256 lbs. be carried for the same money?

7. If 162 *yens*, Japanese gold pieces worth $.99\frac{7}{10}$ cts., pay for a bale of goods, how many rupees, each worth $43\frac{6}{10}$ cts., would have paid for the same?

Compound Proportion.

401. Whenever two ratios separately affect a third term, their total effect is represented by their product. The product of the simple ratios formed by considering like numbers, two and two, is a divisor of the third term. (See *Solution B*, in Simple Proportion.)

If 42 men can build a wall 100 feet long, 20 feet high, and 4 feet thick, in 4 days, working 10 hours a day, *how many*

men must be engaged to build another wall 600 feet long, 18 feet high, and 6 feet thick, in 3 days, working 8 hours per day?

As five conditions affect the result, each may give rise to a problem in Simple Proportion.

I. If 42 men build a wall 100 feet long, how many will be needed to build one 600 feet long?

Statement: 100 : 600 : : 42 : ( )

II. If 42 men are required to build a wall 20 feet high, how many must work to make a wall 18 feet high?

Statement: 20 : 18 : : 42 : ( )

The next three statements would plainly be as follows:

III. *Statement:* 4 : 6 : : 42 : ( ).

IV. *Statement:* 3 : 4 : : 42 : ( ). (Ratio inverse.)

V. *Statement:* 8 : 10 : : 42 : ( ). (Ratio inverse.)

Compounding these ratios—

$$\left.\begin{array}{r@{\;:\;}r} 100 & 600 \\ 20 & 18 \\ 4 & 6 \\ 3 & 4 \\ 8 & 10 \end{array}\right\} : : 42 : (\ ). \quad \textit{Ans.}\ 567 \text{ men.}$$

$$\cancel{6}\,3 \quad 9 \quad 3 \qquad\qquad 1\,\cancel{4}\,7$$

$$\text{For } \frac{\cancel{600}}{\cancel{100}}\times\frac{\cancel{18}}{\cancel{20}}\times\frac{\cancel{6}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{3}}\times\frac{\cancel{10}}{\cancel{8}}\times\cancel{42}=567.$$

402. Rule for Compound Proportion.—*Write for a third term that quantity which is like the answer required.*

Take any two quantities of the same kind and place them as first and second terms, in accordance with the rule for Simple Proportion.

Proceed with all the other pairs of numbers in the same way.

A. *Multiply the third term by the product of all the second terms, and divide this result by the product of all the first terms. Or,*

B. *Divide the third term by the product of the ratios of the couplets.*

EXERCISE CXXXI.

1. A Nevada miner had a quantity of pure silver, and also a mass of pure gold. The silver, when cast, filled a mould 25 inches long, 8 inches wide, and 4 inches deep; his gold filled a mould half as long, two thirds as wide, and six inches deep. A cubic inch of silver weighs .3788 lbs. avoir., and 371¼ grains are worth \$1. A cubic inch of gold weighs .6965 lbs. avoir. An ounce of gold is worth 15.99 times as much as an ounce of silver. What were the respective values of his silver and his gold? Silver, \$5713.88; Gold, —— ?

2. If 7 horses be kept 21 days for \$72, how many may be kept 12 days for \$64?

3. If it takes 1200 yards of cloth, ¾ wide, to make uniforms for 345 soldiers, how much cloth, 5/4 wide, will clothe 805 soldiers?

4. If 1 man can do as much as 2 women, or as much as 3 boys; and if 9 men, 15 women, and 18 boys finish a certain work in 208 days, in what time will 15 men, 12 women, and 9 boys finish the same?

5. If a plot of ground 125 feet 6 inches long by 46 feet 8 inches wide be sold for \$4400, how much will a lot 464 feet 10 inches long and 13 feet 4 inches wide cost?

6. A contract is taken to complete an embankment 11 15/22 miles long in 4 weeks, and 500 men are set to work upon it. At the end of 1 week but 1600 yards are finished. How many additional men must be engaged in order to fulfil the agreement? *Ans.* 1475 men.

7. If 6 oz. of bread cost 4½ cents when wheat is worth \$1.25 a bushel, how much may be bought for 75 cents when wheat costs \$1.62½ a bushel?

8. If the interest on \$126 for 2 years and 6 months at 7% is \$12.05, what is the interest in dollars on £482 @ 5% for 5 years and 3 months, the £ being reckoned at 4.8665?

ANALYSIS.

403. **Analysis** is the solution of problems by the use of *common sense* in the application of general principles.

404. *It is customary to reason from the given number or numbers to* 1 ; *and from* 1 *to the number sought.*

Every problem in Interest and Proportion may be readily solved by Analysis.

The following questions are intended to exemplify various methods of reasoning. If a person have a complex problem *like* one of these, he may, of course, apply the corresponding solution. In all matters of importance it is wise to apply **two** methods of calculation.

Exercise CXXXII.

1. What will 11 sheep cost, if 6 cost $18 ? 1 sheep will cost $\frac{1}{6}$ of $18, or $3 ; and 11 sheep will cost $3×11, or $33.

2. If a loaf weighs 18 oz. when flour costs $10 a barrel, what should a loaf weigh when flour costs $12.

If flour cost but $1 per bbl., instead of $10, the loaf might be 10 times as heavy, and thus weigh 180 oz. If, with flour at $1 a bbl., a loaf weigh 180 oz., at $12 a bbl. it should weigh but $\frac{1}{12}$ as much, or 15 oz. ∴ If a loaf weighs 18 oz. when, etc.

3. Into an empty cistern is turned a stream which would fill it in 2 hours, but for a certain leak, which, if the cistern were full, would empty it in 10 hours. In what time will the cistern fill ?

In one hour the cistern is $\frac{1}{2}-\frac{1}{10}$, or $\frac{2}{5}$, full. To fill $\frac{5}{5}$ at this rate, will require $\frac{5}{5}$ hours$\div\frac{2}{5}=\frac{5}{2}$ hours, or $2\frac{1}{2}$ hours.

4. A boy sold a goat and a wagon for $16. One tenth of the price of the goat was equal to one-sixth of the price of the wagon ; what did he receive for each ?

If the price of the goat is considered as made up of 10 parts, and the price of the wagon as made up of 6 parts, then 1 part of the one will equal 1 part of the other. The 10 parts of the price of the goat, plus the 6 parts of the price of the wagon, taken together, equal \$16. 1 part, then, is worth \$1; the 10 parts of the goat's price are equal to \$10; and the 6 parts, representing the value of the wagon, are equal to \$6.

5. A farmer paid for a stove \$16, which was $\frac{5}{7}$ of what he received for a load of hay; how much did he receive for the hay?

First. If \$16 be 5 parts, what is 1 part?

Second. If $\frac{1}{7}$ of some number be $\$\frac{16}{5}$, what are $\frac{7}{7}$ of the number? $\frac{7}{7}$ of $\$\frac{16}{5}=\$\frac{112}{5}$, or $\$22\frac{2}{5}$.

6. 5 bu. of rye and 3 bu. of oats cost \$4.94, 1 bu. of rye and 4 bu. of oats cost \$2.62. What will 1 bu. of oats cost?

5 times the *price* of (1 bu. of rye and 4 bu. of oats) is the value of 5 times (1 bu. rye and 4 bu. of oats); $(\$2.62 \times 5)=$ value of 5 bu. of rye and 20 bu. of oats.

From (5 bu. rye + 20 bu. oats) = \$13.10,
take (5 bu. rye + 3 bu. oats) = 4.94,

and there remains 17 bu. oats, valued at \$8.16. 1 bu. oats is worth $\frac{\$8.16}{17} = 48$ cents.

7. A hare starts 50 leaps before a dog and takes 4 leaps to the dog's 3; but 2 of the dog's leaps are equal to 3 of the hare's; how many leaps must the dog make to overtake the hare?

Solved thus: If 2 dog-leaps = 3 hare-leaps, 1 dog-leap = $1\frac{1}{2}$ hare-leaps, and 3 dog-leaps will equal $4\frac{1}{2}$ hare-leaps. The dog takes 3 leaps while the hare takes 4, and $\therefore$ gains $\frac{1}{2}$ a hare-leap by so doing; and when the dog makes 6 leaps, he will gain 1 hare-leap. As the hare is 50 leaps in advance, the dog must make (6×50) leaps to overtake the hare.

8. A dealer bought a barrel of spirits for \$30. After diluting it with water, he sold the mixture at a dollar a gallon, and

realized a profit of 10 cents on each gallon purchased. How many gallons of water did he use?

10 cts. $\times 31\frac{1}{2} =$ \$3.05. \$30 + \$3.05 or \$33.05 = the amount realized. As the mixture brought \$1 a gallon, 33.05 gallons were sold. As he bought 31.5 gallons, he must have added the difference, or 1.55 gallons.

9. A steamer has 180 miles the start of another, and sails 5 miles to the other's 8. At what distance from their port of clearance will the second overtake the first?

To gain 3 m. the 2d must sail 8 m.; $\therefore$ to gain 1 m., she must sail $\frac{8}{3}$ m.; and to gain 180 m., the 2d must sail $\frac{8}{3}$ m. $\times$ 180, or 480 miles.

10. Express $69\frac{1}{2}$ miles in metres.

A metre = 3 ft. $3\frac{3}{8}$ in., or $3\frac{27}{96}$ ft.; therefore, 32 metres = 35 yds., or 1 yd. = $\frac{32}{35}$ of a metre. $1° = 1760$ yds. $\times\ 69\frac{1}{2} =$ (139×880) yds. $= \left(\frac{139 \times 880 \times 32}{35}\right)$ metres $= 111835\frac{3}{7}$ metres.

11. Express 11 hhds. in litres.

100 litres = 22 gals. (more exactly, 10000 l. = 2201 gal.) $\therefore$ 11 hhds. $= (\frac{100}{22} \times 63 \times 11)$ litres = 3150 litres.

12. Express 3969 lbs. in kilograms.

200 kilos. = 441 lbs. (as generally reckoned, $2\frac{1}{5}$ lbs. = 1 kilo.)

3969 lbs. $= \left(\frac{200 \text{ kilos.}}{441}\right) \times 3969 = 1800$ kilos.

13. Express a square mile in square metres.

$$\frac{1760 \times 1760 \times 1000}{1196} = \frac{17600000 \times 44}{299} = 2589966.555 \text{ sq. m.}$$

14. If 16 francs be reckoned equal to \$3, how many dollars in 4800 francs?

Ans. \$3 $\times \frac{4800}{16} =$ \$900. For as often as 16 francs are found in 4800, so often the value of \$3 will appear.

15. If $\frac{1}{4}$ of a T. of hay cost £$\frac{2}{5}$, how many crowns will buy $\frac{5}{6}$ T.? (5s. = 1 crown). 1 T. will cost £$\frac{8}{5}$, or 20s. $\times \frac{8}{5}$, which $= \frac{160}{5}$s. $\frac{5}{6}$ T. will cost $\frac{160}{5}$s. $\times \frac{5}{6} = \frac{160}{6}$s. This reduced to crowns $= \frac{160}{30} = 5\frac{1}{3}$ crowns.

16. A teacher gave to each of his pupils 2 books, and had

42 books remaining. If he had given 4 to each, he would have retained but 14. How many pupils had he?

The difference between 42 and 14=28. 2 × the number of pupils = 28. There were, therefore, 14 pupils.

17. If 5 men or 7 women can do a piece of work in 37 days, in what time will 7 men and 5 women do twice as much work?

If 5 men's work = that of 7 women, 1 man's work = $\frac{7}{5}$ of a woman's, and 7 men can do as much as $\frac{49}{5}$ women, ∴ the work of 7 men and 5 women = $(\frac{49}{5}+5)\times 1$ woman's work, or $\frac{74}{5}$ times 1 woman's work.

18. A tree in falling was broken into two pieces. The total length was 98 feet. If the shorter piece was $\frac{2}{5}$ of the length of the longer, how long was the shorter piece?

(In questions similar to this, assume the longer or larger as a unit.)

The length of the shorter, $\frac{2}{5}$, + the length of the longer, $\frac{5}{5}$, in all 7 parts equal 98 feet. 1 part=14 feet, and 2 parts, *i.e.*, the $\frac{2}{5}$ representing the shorter piece,= 28 feet.

19. What o'clock is it, if the time from now till midnight will equal 3 times the number of hours since noon?

Solved thus: Divide the twelve hours between noon and midnight into four equal parts. One of these parts = 3 hours: The other three parts are of course equal to three times this one part. Then the time is 3 hours past noon, or 3 o'clock.

20. A man was hired for a period of 40 days, and was to receive \$2 for every day's labor; but for every idle day he agreed to forfeit \$1 besides the day's wages. At settlement he received \$50. How many days was he idle?

Solved thus: If he had lost no time, he would have received \$80. His idleness cost him \$30. Each idle day entailed a loss of \$2 wages and \$1 forfeit, or \$3 for 1 lost day. To lose \$30 he must have been idle 10 days.

21. Alfred had 5 cakes, and John had 3. Henry lunched with them, and paid 8 cents for his portion. How many cents should Alfred receive?

Alfred, in partaking of the lunch, eat $2\frac{2}{3}$ cakes himself, John also eat $2\frac{2}{3}$ cakes. Of the $2\frac{2}{3}$ which Henry eat, $2\frac{1}{3}$ were from Alfred's stock, and only $\frac{1}{3}$ from John's. Therefore, Alfred should be paid 7 cts. for $\frac{7}{3}$, and John 1 ct. for his $\frac{1}{3}$ of a cake.

22. B., G., and W. (Brown, Green, and White) are boat-builders. When B. and G. work together, they can build a boat in 4 days; B. and W. require 5 days; G. and W., 6 days. In how many days can each build a boat?

If B. and G. work together 1 day, and then B. and W. a day, and then G. and W. a day, $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$, or $\frac{37}{60}$, of one boat will have been built. As each has worked 2 days, $\frac{37}{120}$ will represent what B., G., and W. can together do in 1 day. As G. and W. can do $\frac{1}{6}$ or $\frac{20}{120}$ in a day, $\frac{37}{120}-\frac{20}{120}$, or $\frac{17}{120}$ = what B. can do in 1 day. B. will therefore require $\frac{120}{17}$, or $7\frac{1}{17}$ days, to build a boat. Like reasoning will show that G. in 1 day can do $\frac{37}{120}-\frac{24}{120}$, or $\frac{13}{120}$; and that he will require $9\frac{1}{40}$ days; and also that W. can do $\frac{37}{120}-\frac{30}{120}$ or $\frac{7}{120}$ in a day, and will therefore build a whole boat in $\frac{120}{7}$ or $17\frac{1}{7}$ days.

23. The head of a fish is 9 inches long, its tail is as long as its head and half its body; and the length of its body equals the length of its head and tail taken together.

Head.	Body.	Tail.
9 in.	$\frac{1}{2}$	$\frac{1}{4}+9$ in.

$\frac{1}{2}+\frac{1}{4}+9$ in.$+9$ in.$=\frac{4}{4}$. $\frac{1}{2}+\frac{1}{4}+18$ in.$=\frac{4}{4}$ $\therefore$ $\frac{1}{4}=18$ in. and $\frac{4}{4}=$ 72 in., or the whole length of the fish.

24. Distribute 48 cents among three boys so that their shares shall be as 1, 5, and 6.

Analysis: As the second has 5 shares like that of the first, and as the third has 6 such shares in all, there must be 12 equal shares. One of twelve equal parts of 48 cents will equal 4 cents; five such parts will equal 20 cents; and six such parts will equal 24 cents. These sums will represent the respective shares of the three boys.

25. If E., F., and G. receive $21.40 for doing a piece of work at which E. has labored 8 hours a day for 4 days, F. 5

hours a day for 3 days, and G. 10 hours a day for 6 days, what should each receive?

Analysis: As 107 represents the hours of labor, $21.40 may be divided into 107 parts. E. will receive 24 parts, F. 15 parts, and G. 60 parts.

NOTE.—For **class drill**, see *Sections on Proportion*.

INVOLUTION AND EVOLUTION.

405. Powers are the **products of equal factors.** *Second powers* are products of two equal factors. *Third powers* products of three equal factors, *fourth powers* of four equal factors.

406. The number from which a power is derived is called the **root.** The class or degree of the power is indicated by a small figure, above and at the right of the root, called an **index** or **exponent.** $5\times5=5^2$; $5\times5\times5=5^3$, equal the second and third powers of 5; read, five second power, and five third power.

407. The root is sometimes called its own **first power**; the second power is commonly called the **square**; the third power the **cube**, as the product of its two dimensions represents the surface of a square, and of its three dimensions, the contents of a cube.

408. Involution is the process of forming the powers of numbers. **Evolution** is the process of resolving powers into their equal factors.

409. TO FIND A POWER.

Employ the root as a factor as many times as there are units in the exponent.

Exercise CXXIII.

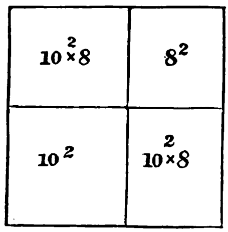
1. Square 4.	6. Find 7^5.	11. Square $\frac{1}{15}$.
2. Cube 5.	7. Find $(\frac{2}{3})^4$	12. Cube $\frac{2}{7}$.
3. Square $\frac{1}{5}$.	8. Find $\frac{3^4}{2^5}$.	13. Cube 456.
4. Cube $\frac{2}{5}$.	9. Find 8^3.	14. Square 1241.
5. Find 5^4.	10. Find 6^3.	15. Cube $41\frac{1}{2}$.

Memorize the following **Table of Powers:**

First Powers,	1,	2,	3,	4,	5,	6,	7,	8,	9.
Second Powers,	1,	4,	9,	16,	25,	36,	49,	64,	81.
Third Powers,	1,	8,	27,	64,	125,	216,	343,	512,	729.

410. *A number may be squared by first separating it into two parts, and then multiplying each of the parts by each of the parts, and adding the products. Axiom VII.*

$$\begin{array}{ll} 18\times18 & =(10\times\ 8)\times(10\times8) \\ (10+\ 8)\times\ 10 & =10^2+\ 10\times8 \\ \underline{(10+\ 8)\times\ \ 8} & =\underline{\qquad\quad 10\times8\ +8^2} \\ \therefore 18\times18 & =10^2+2(10\times8)+8^2 \\ \quad 324 & =100+\ \ 160\ \ +64 \end{array}$$



PRINCIPLE I.

If a number consists of two parts, the square of that number consists of the square of the first part, plus twice the product of the first part by the second, plus the square of the second part.

By inspecting the squares of the following numbers it will be found that—

PRINCIPLE II.

—*The square of any number must contain at least twice as many figures, less one, as the root itself.*

1,	10,	100,	1000,	9,	99,	999.
1,	100,	10000,	1000000,	81,	9801,	998001

411. Evolution is indicated by the **radical sign,** $\sqrt{\ }$. The root to be found is shown by the figure in the angle. $\sqrt[3]{\ }$ indicates the 3d or cube root; $\sqrt[4]{\ }$, the 4th root.

412. Evolution may also be shown by means of a **fractional exponent,** the denominator indicating the root. Thus:

$$\sqrt[3]{8} = 8^{\frac{1}{3}} = 2\,; \quad \sqrt{25} = 25^{\frac{1}{2}} = 5\,; \quad 16^{\frac{1}{4}} = 2.$$

Square Root.

413. As tens into tens produce hundreds, and not units nor tens, if there be three or four figures in a second power, the third and fourth orders will give rise to a root figure of the second order. The root of figures standing in the fifth and sixth orders will be one or more hundreds. (*Principle II.*)

Again, from Principle I. we infer, that—

If the square of the tens of a root be subtracted from the hundreds of a power, there will remain twice the product of the tens into the units, plus the square of the units.

$$\overline{416}^2 = 173056$$
$$= (400+10+6)^2 \quad \text{Ax. VII.}$$

$$\begin{array}{rrrrrr}
 & & & & 400+10+6 & \\
 & & & & 400+10+6 & \\
\hline
 & & & 400\times6 & 10\times6 & 6^2 \\
 & 400\times10 & 10^2 & 400\times6 & 10\times6 & \\
400^2 & 400\times10 & & & & \\
\hline
\end{array}$$
$$400^2+2(400\times10)+10^2+2(400\times6)+2(10\times6)+6^2$$

Explanation: Since there are 3 groups of two figures, there will be 3 root figures. The square of a third figure will always be found in the fifth order. The root of the third period will be 4. This being subtracted, the remainder will consist principally of the product of the second figure into twice the first. We

$$\begin{array}{r|r}
 & \sqrt{17,30,56}\;)\underline{416} \\
 & 16 \\
\hline
80 & 130 \\
1 & \\
\hline
81 & 81 \\
\hline
820 & 4956 \\
6 & \\
\hline
826 & \underline{4956}
\end{array}$$

add a naught to twice the first, since it stands as tens to the second figure, and use this sum as a divisor. Having found the second figure, we add it to the divisor, in order to count in its square for subtraction. After this subtraction, the remainder consists chiefly of twice the first, plus twice the second, into the third figure. We therefore double the root already found, and annex a cipher to occupy the place of the third figure until it is found. Having discovered this, we add it to the trial divisor in order the more conveniently to subtract its square from the last dividend; for when it is multiplied along with the rest of the divisor, it is squared.

NOTE.—The teacher should first solve a few examples; then have the first three sections of the rule memorized, and a few easy problems worked out. Afterward, explain the rule in accordance with the preceding principles.

TO EXTRACT THE SQUARE ROOT.

RULE I.—*Separate the figures into periods of two figures each, beginning with the units.*

II. *Find the greatest square in the left-hand period, and, after writing its root in the quotient, subtract the square, and to the remainder annex the next period for a dividend.*

III. *Annex a cipher to twice the root already found for a trial divisor; write the quotient as a second root figure and also add it to the trial divisor to complete it: multiply the complete divisor, subtract the result, and, to the remainder, annex the next period on the right.*

IV. *Proceed thus till every period has been annexed. If there be a remainder, the decimal figures of the root may be found by annexing two ciphers after each successive division.*

V. *If any dividend be too small to contain the divisor, a cipher must be placed in the quotient and the next period be annexed. (Sometimes a remainder is slightly greater than the divisor.)*

NOTE.—If a dividend will not contain the divisor, write a cipher as a root figure and annex another period. To find the root of a fraction whose terms are not perfect squares, first reduce the fraction to a decimal form.

414. Perhaps a teacher having a young class would do well to proceed somewhat as follows:

"Let us find the two equal factors of 324. One of these is its *square root.* As 324 consists in part of hundreds, its root must be one or more tens. It cannot be so much as 2 tens, for the square of 2 tens is 400. 324 must, therefore, consist of $10^2+2\times(10\times?)+?^2$ Let us proceed to analyze it regularly:

$$
\begin{array}{rlrcl}
 & & 324 & (10+8 & \\
 & 10^2 & 100 & & \\
\hline
2\times10= & 20 & 224 & = & 2\times(10\times\ ?)+?^2 \\
 & 8 & & & \\
2\times10+8= & \overline{28} & \underline{224} & = & 2\times(10\times8)+8^2.
\end{array}
$$

"Having subtracted the square of the first, the remainder must consist of twice the first into the second. We know the first; we may therefore double it and use it as a divisor. Then, by trial, we may find the other factor. 20 is contained in 224 say 8 times, for we must allow for the subsequent deduction of the third element; viz., the square of the last figure. Now, 8 is not only a multiplier of 2×10, but also of itself, therefore we will add it to twice 10, and multiply the sum by 8."

415. A **perfect square** is one whose exact root can be found. If the square root of an integer be not an integer, it cannot be *accurately* determined. The following are not *perfect squares:* an even number not divisible by 4; a number terminating in 2, 3, 7, or 8; a number whose units figure is 5, unless its tens figure be 2; a number having an odd number of ciphers. If a fraction has been reduced to its lowest terms, its square root must be a fraction. Imperfect squares are called surds, or irrational numbers, because their relation to other numbers cannot be accurately expressed.

Exercise CXXXIV.

Extract the square root of the following :

1–6. 625; 698485; 6084; 4906; 841; 1287.

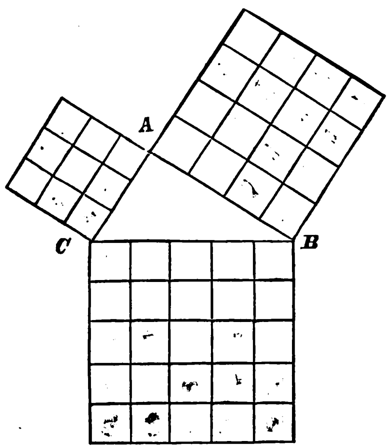
7–9. 56821444; 714025; 9585216.

10–13. 167.9616; 28.8369; 57648.01; .042849.

14–20. $\frac{3}{5}$; $\frac{1}{17}$; $2\frac{1}{6}$; $\frac{3\frac{1}{2}}{4\frac{1}{2}}$; $\frac{2}{7}$; $\frac{4.15}{2.63}$; $1\frac{56}{169}$.

Note.—Reduce each of the last 7 to a decimal, and if the root does not sooner terminate, use at least 3 decimal periods. Thus, $\frac{3}{5}$=.60000. $\sqrt{.60000}$=.774+

416. The applications of the rule for extracting the square root are many and valuable. Most of these will appear under the head of Mensuration. Some of the most important depend on this theorem in Geometry, viz.: that the square described on the long side of a right-angled triangle will equal the sum of the squares described on the other two sides.



Thus, if AB is known and also AC, BC can be found by extracting the square root of the sum of the squares of AB and BC. Let AC be 3, then AC^2 will be 9; and let AB be 4, then AB^2 will be 16; then will BC^2=25, and $\sqrt{BC^2}$, or BC=5.

Let AB be 20 and AC be 15, then will BC be 25, for $\sqrt{20^2+15^2}=25$. Thus $20^2=400$, $15^2=225$; $\sqrt{625}=25$.

Cube Root.

417. TABLE OF THIRD POWERS.

$2^3=8$, $3^3=27$, $4^3=64$, $5^3=125$, $6^3=216$, $7^3=343$, $8^3=512$.

$1^3=$	1	$.1^3$	$=.001$
$9^3=$	729	.9	$=.729$
$10^3=$	1000	.01	$=.000001$
$99^3=$	970299	.99	$=.970299$
$100^3=$	1000000	.001	$=.000000001$
$999^3=$	997002999	.999	$=.997002999$
$1000^3=$	1000000000		

418. The cube of a number cannot contain more than three times as many figures as the number itself, nor less than 3 times as many, less two. Hence

I. *If the figures of a power be separated into periods of three figures, there will be as many figures in the root as there are periods.*

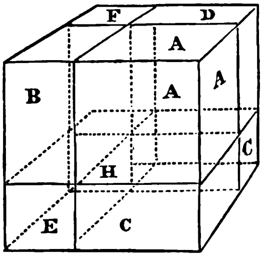
419. II. *If a number be divided into two parts, the cube of that number equals the cube of the first part plus three times the square of the first multiplied by the second; plus three times the square of the second multiplied by the first, plus the cube of the second.* (Axiom VII.)

This may be shown thus: Suppose 10 to be separated into the two parts, 7 and 3, then 10^3, or 1000, will equal $(7+3)^3$.

$$\begin{array}{rrrr}
 & & & 7+3 \\
 & & & 7+3 \\
\hline
 & & & 21+9 \\
 & & 49+ & 21 \\
\hline
 & & 7^2+2\times(7\times3)+ & 3^2 \\
 & & & 7+3 \\
\hline
 & & 7^2\times3+2\times(7\times3^2)+ & 3^3 \\
 & 7^3+2\times(7^2\times3)+ & 7\times3^2 & \\
\hline
 & 7^3+3\times(7^2\times3)+ & 3\times(7\times3^2)+ & 3^3 \\
10^3=1000=(343+ & 441+ & 189+ & 27)
\end{array}$$

$15^3=3375=(10+5)^3$

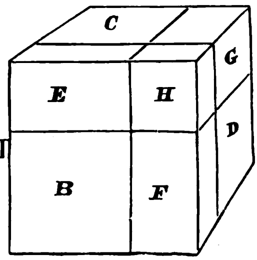
I



$$
\begin{array}{r}
10+5 \\
10+5 \\
\hline
10\times5+5^2 \\
10^2+\quad 10\times5 \\
\hline
10^2+2\times(10\times5)+5^2 \\
10+5 \\
\hline
10^2\times5+2\times(10\times5^2)+5^3 \\
10^3+2\times(10^2\times5)+\quad 10\times5^2 \\
\hline
10^3+3\times(10^2\times5)+3\times(10\times5^2)+5^3 \\
3375=1000+\quad 1500\quad+\quad 750\quad+25
\end{array}
$$

Figures I. and II. may represent two views of a cube of 15 inches. If divided as shown, eight parts will be formed which will correspond to the products in the text. $A^3 = 10'$ $B=10^2\times5$, $C=10^2\times5$, $D=10^2\times5$, $E=10\times5^2$, $F=10\times5^2$, $G=10\times5^2$ and $H=5^3$.

II



Let n stand for any number, and let t stand for one part of it, and u for the remaining part, then will

$n^3=t^3+3\times t^2\times u+3\times t\times u^2+u^3$.

$17^3=9^3+(3\times9^2\times8)+(3\times9\times8^2)+8^3$.

$4913=729+1944+1728+512$.

Again: let $n=24$, $t=20$, and $u=4$.

$24^3=20^3+(3\times20^2\times4)+(3\times20\times4^2)+4^3=13824$.

$17^3=1000+300\times7+3\times490+343=$? Why?

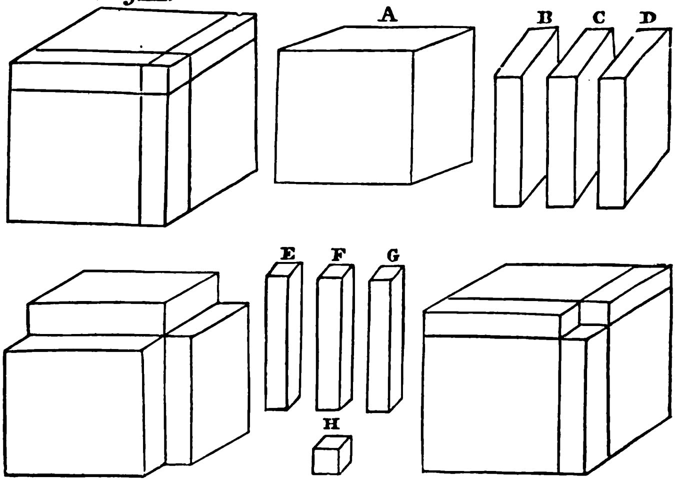
$61^3=21600+3\times3600+180+1=$? Why?

$2\frac{1}{4}^3=$? $(\frac{3}{5})^3=$? $(\frac{7}{12})^3=$? $(\frac{6}{13})^3=$? $4.2^3=$? $3.7^3=$?

The cube of $\frac{3}{5}=\frac{27}{125}$. The cube of $4.2=42^3\div1000$.

The facts already presented, together with those referred to in connection with the following figure (*Fig. III.*), should be brought clearly before a class before proceeding to the actual work of evolution. Reference may be made to the correspondence of Multiplication and Division, page 20, by way of exemplifying what it is desired to show.

Fig. III.



Let Figure III. represent a cube, the volume of which is 15625 cubic inches. It may be constructed of eight such parts as A, B, C, D, E, F, G, and H. A will represent the cube of the tens figure; the other parts may be found as follows:

OPERATION:

			15,625(25
The cube A, ...	2^3		8.
			7625
Trial divisor formed, . .	$2^2 \times 300 = 1200$		
Complement numb. formed,	2×3, 6, 65, 65×5,	325	
Complete divisor formed, . . .		1525	7625

Explanation: As the edge of A must be more than 10 inches and less than 30 inches, the tens figure is 2; and therefore the volume of A equals 8000 cubic inches. When A is deducted from the figure, there remain 7625 cubic inches. These are found, for the most part, in the additions B, C, and D, each of which corresponds in length and width to one side of A. The area of these three is 1200 square inches, or the

trial divisor. This question then presents itself: "What is the thickness of a body containing nearly 7625 cubic inches, if its surface equals 1200 square inches?" The answer is, plainly, "5 inches." As the width of the narrow quadrangular prisms equals this thickness, the surface of the three prisms E, F, and G, and that of the small cube, can be computed. It equals their length when united multiplied by the width or thickness. This product equals the **complement number**—and added to the trial divisor makes the **complete divisor.**

SUMMARY.

420. In explaining the following rule, by referring to the geometrical figures, it should be shown that the **trial divisor** is equal to the sum of the *three surfaces* of the solids which match the leading cube. The quotient figure is equal to the thickness of either of these. If one side of each of the *four* smaller additions be taken and these be added, the sum will represent **the complement number.** The sum of the complement number and the trial divisor will be equal to **the complete divisor**—that is, to the united *area* made by taking one side of each of the *seven* additions made to the leading cube.

421. TO FIND THE CUBE ROOT OF A NUMBER.

Point off the given number into periods as if in notation.

Find the greatest cube in the left-hand period, and write its root as a quotient. Subtract the cube from the left-hand period, and to the remainder annex the next period for a dividend. Consider the root figure already found as tens, and take three times its square for a TRIAL DIVISOR, *with which find a second root figure (taking care not to make too high an estimate). Annex the figure last found to three times the preceding part of the root, multiply by the last figure, and add this* COMPLEMENT NUMBER *to the trial divisor to complete it. Multiply the* COMPLETE DIVISOR *by the last root figure, and, after subtracting the product from the dividend, annex the next period to the remainder for a new dividend. To find subsequent trial divisors, add together the last complement*

number, the last complete divisor, and the square of the last root figure, and to the sum annex two ciphers.

To make it complete, proceed as before. Continue till all the periods have been annexed.

Proof.

Raise the root to the third power, adding the remainder, if any, to the result.

1. What is the cube root of 162784.5 ?

5	7500	162784.500(54.601(4)
3	616	125
154	8116	37784
4	16	32464
616	874800	5320500
54	9756	5307336
3	884556	13164000000
1626	36	8943643801
6	8943480000	4220356189
9756	163801	
5460	8943643801	
3		
163801		

Explanation.

The 1st trial divisor is 7500; the 2d trial divisor, 874800; the 3d, 8943480000. The 1st complement number is 616; the 2d, 9756; the 3d, 163801.

Note.—In finding the cube root of a *common fraction*, first reduce it to a decimal, and treat the result as if it were an integer. The number of decimal places in *the power* should be three, or some *multiple* of three. The number of decimal *root* figures will equal the number of decimal *periods*.

Exercise CXXXV. a.

Find the cube root of each of the following :

1–7. 259694072; 15,625; 95,443,993; 175,616; 48,228,550; 7.000000000; 1157.625.

8. The product of 3 equal numbers is 32768; what are they?

9. What is the length of each of the equal sides of a block of stone which contains 56 solid feet and 568 solid inches.

10. If a mass of ice weigh 12859791222.5 lbs., how large a cube would it form? (57.5 lbs.=1 cu. ft.)

11. A cube of cast-iron weighs 352 tons. How long is its edge, if a cubic inch weigh .2607 lb.?

12. A cubical reservoir 5 times as long as it is wide, and 5 times as wide as it is deep, contains 62500 tons of water. Allowing $62\frac{1}{2}$ lbs. to the cu. ft., find its dimensions?

NOTE.—Consider the depth a unit, and the whole will appear as though made up of 125 cubes.

13. A pile of ore whose length, width, and thickness were equal was sold for $34560, it being estimated at 1 cent per cubic inch. How high was the pile?

422. Similar solids are to each other as the cubes of their like dimensions. Thus: A box 4 times as long as another of the same shape, will hold 64 times as much as that other. A ball whose diameter is 3 times as great as that of another ball of the same material, will weigh 27 times as much. If the larger of two fish which are shaped alike be $2\frac{1}{2}$ times as long as the smaller, he should weigh nearly 16 times as much as the smaller.

EXERCISE CXXXV. b.

1. If a 32-lb. ball have a diameter of 6 in., what is the diameter of a 4-lb. ball?

2, 3. **A ball equals .5236 of the cube whence it is turned.** A cu. in. of cast silver weighs .3788 of a lb. avoir. $371\frac{1}{4}$ grains are worth $1. How many spheres will 342730.957 lbs. of silver make, and what is it worth?

4–7. What is a ball of pure silver worth whose diameter is 2 inches? Whose diameter is 5 inches? Whose circum-

ference is 25.1328 inches? What will the last ball weigh? A circumference = 3.1416 diameters.

8. **A pyramid equals $\frac{1}{3}$ of a cube having an equal base.** Cheops is 520 feet high and covers eleven acres. What would be the height of a cube one-millionth as great in bulk?

9. If a bushel represents a cube of 12.9 inches, what cube does 30 bushels equal?

10. If David measured 5 ft. 10 in. in height and weighed 175 lbs., what did Goliath weigh, his stature being 11 ft. 4 in.?

Ans. 1283.39 lbs.

11. Four men having bought a ball of silk 100 cubic inches in volume, proceed to share it equally. To what extent is the diameter diminished as each winds off his share?

423. Questions on Powers and Roots.

Define the terms Power, Root, Involution, Evolution, Square, Cube, Exponent. Repeat Axiom VII. and show how it is applied in the formation of the square of 7+3, and the square of 10+5. How many figures must be written to express the square of a number of two figures? Of three figures? Of four figures? What use is made of the knowledge of these facts. What will be the first digit in a square root? How is the second digit found? How is the square root of a fraction found? How is a third power formed? How many figures may it contain in proportion to the root number? How many blocks are required to illustrate cube root, and what are their forms? What do the divisors in the rule for cube root actually represent? What will volume divided by surface give?

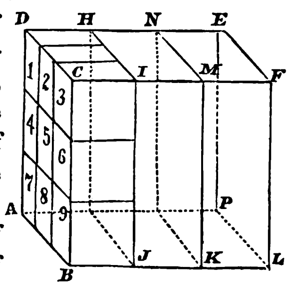
Why do we triple the value of the root already found? How are similar solids proportioned to each other? Give four illustrations referring to objects of dissimilar shapes. What is the process of involution? (*Ans.* Synthesis.) What is the process of evolution? (*Ans.* Analysis.) What is an integer called when it is not a perfect square?

MENSURATION.

424. Before proceeding to the measurement of surfaces and solids, certain descriptive words and phrases should be clearly comprehended. Among the most important of these are the following: **Line**—straight, curve, vertical, oblique, horizontal, and perpendicular; **angle**—right, obtuse, acute; **rectangle**; **triangle**—equilateral, isosceles, right-angled; **circle**—circumference, centre, radius, diameter, arc, chord, sine, degree, area; **solid**—cube, prism, cylinder, cone, pyramid, sphere, spheroid, frustum, segment, together with the terms surface, altitude, perimeter, base, vertex, volume, apothegm, and tangent.

The columns of a dictionary, the pages of a geometry, or the introductory chapters of a good geography will afford fair explanations of most of these words. The Appendix of this volume may be referred to for brief definitions. *Pupils should write them out, and then memorize them.*

Pupils should be impressed with definite notions of a foot, a yard, a square foot, a square yard, three square feet, and three feet square, etc. In drawing small objects, endeavor to make correct measurements, in order, if possible, to accustom the pupil to the *actual* dimensions of objects. It would also be well if the metre were carefully defined and contrasted with the yard. As a metre equals 39.368505 ins., a kilometre equals about 5 furlongs. A cubic metre equals 35.3166 cubic feet.



The term **volume** may refer either to capacity, contents, or solidity. Refer to tables 14, 15, 16, 17, and 18. Tables 19 and 20 are **nearly related**. The relation of clock-time to longitude may be shown by means of a globe.

Representations of a variety of objects should be made, such as of

loads of wood, blocks of stone, vats, and cellars. D L, the cube on the page preceding, may be explained as a cubic yard, or 27 cubic feet. D J = 9 cubic feet.

425. *An Object Lesson.*

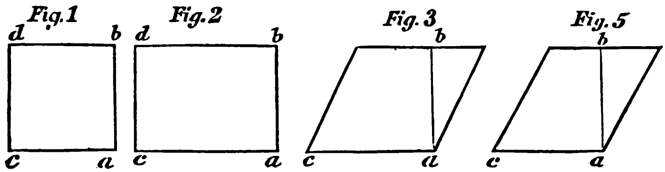
Let the teacher seat his class before a blackboard. Having prepared a looped and knotted cord, a crayon and a ruler, he may proceed as follows: "I will endeavor to describe the different parts of a circle. Observe that I press the knot in this cord firmly against the blackboard, and extend the string by means of the crayon caught in the loop. As I push the crayon forward, notice that the mark curves until it finally reaches the point of departure. This complete curve is called a **circumference.** The space it encloses is called a **circle.** The line of the string is a **radius.** As in forming the curve the string was extended in every direction, there are as many radiuses, or radii, in a circle as one may choose to think of. The point covered by the knot is the **centre.** Every radius extends from the centre to the circumference. A **diameter** is a line extending across the circle and passing through the centre. If two diameters be drawn across each other in such directions as to divide the circle into four equal parts, they will be **perpendicular** to each other. Each of the four parts, or **sectors,** of the circle thus formed is called a **quadrant.** If the circumference be divided into 360 equal parts, a quadrant will be measured by 90 of these parts. Each part is called a **degree** (°). All radiuses extend from the centre. As they diverge from each other they form **angles.** These are greater or less than the angle of a quadrant. The angle at the centre in a quadrant is a **right angle.** Acute angles are less than right-angles; obtuse angles are greater. Any part of a circumference which is cut off by two radii is called an **arc.** A **chord** is the straight line which unites the extremities of an arc. A **segment** is that part of a circle which is bounded by an arc and its chord."

426. ***Mensuration*** is the process of ascertaining the length of lines, the area of surfaces, and the volume of solids.

427. **Units of surface** are considered as squares; units of **volume of solidity**, as cubes.

428. **Parallelograms** have four sides, of which those opposite are parallel.

429. I. **The area of a parallelogram** EQUALS *the product of its length and breadth.*

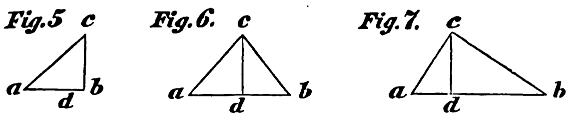


A square has equal sides and equal angles. *Fig.* 1.
A rectangle is an equiangular parallelogram. *Fig.* 2.
A rhomboid has parallel sides, but is not right-angled. *Fig.*3.
A rhombus is an equilateral rhomboid. *Fig.* 4.
In *Figs.* 1, 2, 3, and 4, *ab* represents the altitude.

430. *Triangles.*

II. To compute the Area of a Triangle.

Rule.—*Multiply the base by the altitude and divide the product by* 2. *Fig.* 5, 6, or $7 = (ab \times cd) \div 2$.



III. To compute the Area of a Triangle by the Length of its Sides (*Figs.* 5, 6, and 7).

Rule.—*From half the sum of the three sides subtract each side separately; then multiply the half-sum and the three re-*

mainders continually together, and take the square root of the product.

For economy in space, let $\frac{ab+ac+bc}{2}=m$; *i.e.*, $m=$ sum of 3 sides $\div 2$. Then $\sqrt{(m-ab)\times(m-ac)\times(m-bc)\times m}=$area.

Let $ab=5$, $ac=3$, $bc=4$ (*Fig.* 7). Then, $(5+3+4)\div 2=6$. Then, by the rule, $\sqrt{(6-5)\times(6-3)\times(6-4)\times 6}$, or $\sqrt{36}=6$ $=$area. When the *sides are equal,* the area will be side $\times$ side $\times .433$. Thus, the area of an equilateral triangle whose side is 10 will be 43.3; for $10^2\times.433=43.3$.

IV. To ascertain the **Hypothenuse** or longest side of a Right-angled Triangle.

Rule.—*Find the square root of the sum of the squares of the other two sides.*

Thus, in *Fig.* 5, $\sqrt{ab^2+bc^2}=ac$. If $ab=3$, and $bc=4$, then $ac=5$; for $\sqrt{3^2+4^2}=5$.

V. To ascertain either of the **Shorter Sides** of a Right-angled Triangle.

Rule.—*Subtract the square of the given leg from the square of the hypothenuse, and take the square root of the remainder.*

In *Fig.* 5, if $ab=36$, and $ac=60$, then will $bc=48$; for $\sqrt{60^2-36^2}=48$, as $3600-1296=2304$, and $\sqrt{2304}=48$.

VI. When the Hypothenuse of a Right-angled Triangle having equal Sides is given, to find a side.

Rule.—I. *Divide the Hypothenuse by* 1.414213; or, II. *Extract the square root of* $\frac{1}{2}$ *the square of the Hypothenuse.*

In *Fig.* 5, if $ac=400$, then $ab=282.84$.

VII. The Height of a Triangle equals Twice the Area divided by the Base.

In *Fig.* 8, if $ab=12$, and the area $=60$, then $bc=10$; for $(60\times 2)\div 12=10$.

VIII. To find the Height or Perpendicular when the Base and two Sides are given (*cd*, *Fig.* 6).

Rule.—*As the base is to the sum of the sides, so is the difference of the sides to the difference of the divisions of the base. Add the half-difference to the half-base for the longer division. Then, by Rule V., the square of the longer side (considered as the Hyp. of the larger of the two triangles which make up the given triangle) minus the square of this longer section of the base = the square of the height.*

In *Fig.* 7, if $ab=12$, $ac=6$, and $bc=10$, then will $cd=5.68+$.

Operation. $12:16::4:5\frac{1}{3}$. Then, $\frac{12}{2}+(5\frac{1}{3}\div 2)=bd$ $=8\frac{2}{3}$. Hence $ad=3\frac{1}{3}$. Now, by Rule V., $\sqrt{10^2-(8\frac{2}{3})^2}=4.99$ $=$perpendicular, or cd.

Proof.—In accordance with Rule III. we proceed as follows: Assuming that $ab-ac=6$, and $bc=10$ $(12+6+10)\div 2=14$. $(14-12)\times(14-6)\times(14-10)\times 14=896$. $\sqrt{896}=29.94$ $=$area of *Fig.* 7. Now, by Rule II., $(12\times 4.99)\div 2=29.94$, or, by Rule VII., $(29.94\times 2)\div 12=4.99$.

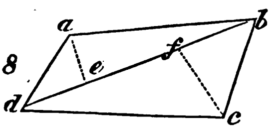
Rule IX.—**The corresponding linear parts of similar figures are in proportion.**

Example.—The shadow of a shrub 4 ft. in height was 6 feet; the shadow of a tree was 90 ft.; how high was the tree?

$6:90::4$: height of the tree, or 60 ft.

431. A ***Trapezium*** is a quadrilateral having unequal sides.

Fig. 8



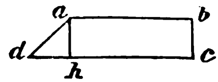
X. To find the Area of a Trapezium.

Rule.—*Multiply the diagonal by the sum of the two perpendiculars falling upon it, and divide the product by 2.*

Example.—The diagonal $db=40$, the perpendicular $ae=8$, the perpendicular $cf=16$. Operation. $40\times(16+8)\div 2=480$.

432. *Trapezoid.*

DEFINITION.—*A quadrilateral with only one pair of sides parallel.*



XI. TO FIND THE AREA OF A TRAPEZOID.

RULE.—*Take half the product of the perpendicular into the sum of the parallel sides.* $(ab+cd)\times h\div 2=area.$

Example.—Let $dc=33$, and $ab=25$, and $ah=15$. Then $(25+33)\times 15\div 2=435=$area.

433. *Polygons.*

DEFINITION.—*Plane figures having three or more sides. They are said to be regular when their sides are equal.*

XII. TO FIND THE AREA OF A REGULAR POLYGON.

434. When the number of sides, the length of a side, and the distance from the centre to the middle of a side are given.

RULE.—*Multiply the length of a side by the perpendicular distance to the centre, multiply the product by the number of sides, and divide this result by* 2.

Example.—What is the area of a hexagon whose side is 6 ft., the distance from the centre being 5.2 ft.

$6\times 5.2\times (6\div 2)=93.6.$

XIII. TO FIND THE AREA OF THE FIGURES NAMED.

3	Triangle	$=\text{side}^2\times .433.$	8	Octagon	$=\text{side}^2\times 4.828.$
4	Square	$=\text{side}^2\times 1.$	9	Nonagon	$=\text{side}^2\times 6.182.$
5	Pentagon	$=\text{side}^2\times 1.72.$	10	Decagon	$=\text{side}^2\times 7.694.$
6	Hexagon	$=\text{side}^2\times 2.598.$	11	Undecagon	$=\text{side}^2\times 9.366.$
7	Heptagon	$=\text{side}^2\times 3.634.$	12	Dodecagon	$=\text{side}^2\times 11.196.$

NOTE.—The data for these rules and for many of those which follow were obtained from Haswell's *Engineer's and Miner's Pocket-Book*, with the author's permission. The volume referred to is a wonderful storehouse of useful facts and principles. These are stated with

accuracy and minuteness. It is a work indispensable to scientific mechanics, architects, and engineers. The same volume will furnish data for describing the circles which contain polygons, or which are inscribed in them; also for determining the length of a side of a polygon whose area is given, or of the radius of the related circle. For instance, by special rule, if the area of a 12-sided figure be 16 inches, the length of a side will be $\sqrt{16} \times .2989$ or 1.1956. The radius of the inscribed circle will equal $\sqrt{16} \times .5577$ or 2.2308.

435. *Irregular Polygons.*

DEFINITION.—*Figures with unequal sides.*

XIV. TO COMPUTE THE AREA OF AN IRREGULAR POLYGON.

RULE.—*Draw diagonals so as to divide the figure into triangles or quadrilaterals; ascertain the areas of these, separately, and take their sum.*

NOTE.—This rule is of great consequence to persons not versed in Trigonometry. Until recently, even surveyors generally computed areas by dividing the field in question into triangles, measuring the sides of these, and applying Rule XIV.

EXERCISE CXXXVII.

1. In a triangular field whose longest side is 60 rods, the shortest distance from that side to the junction of the shorter sides is 30 rods. What is its area in acres?

2. How many square yards in a plot whose sides are respectively 26, 34, and 36 feet?

3. How long is a ladder, if, when the foot is set 15 feet from a wall, the top rests 45 feet above its base?

4. If a ladder 44 feet long just reaches to the top of a vertical pole when the foot of the ladder is set 12 feet from the base of the pole, how high is the pole?

5. The ridge of a double roof having equal rafters is 12 feet higher than the eaves. If each rafter is 40 feet long, how wide is the house?

6. If a room be 22 ft. by 20, and the ceiling be 14 ft. high, what is the length of the longest cord which can be extended in it? $\sqrt{\text{length}^2+\text{breadth}^2+\text{height}^2}$.

7. The base of a triangle is 140 rods; the area is 40 acres; what is the height of the triangle, or, in other words, what is the shortest distance from the junction of the other sides to the base?

8. In a certain triangle the base is 40 feet; one side is 26 ft. long, and the other is 32 ft. in length; what is its altitude—that is, the perpendicular on its longest side?

9. If the longest side of a triangle be 28 feet, and the other two sides be respectively 20 ft. and 18 ft., what is its altitude?

10. The height of a triangle whose base is 12 ft. is 4 ft., what is the height of a similar triangle whose base is 15 ft. 6 in.?

11. The four sides of a trapezium are in length as follows: 6, 8, 11, 11.5 ft., and the two perpendiculars drawn to the diagonal are 3 and 5 ft. respectively. What is its area?

12. The diagonal of a trapezium is 16 feet, and the two perpendiculars drawn to the vertices of the sides are respectively 3 ft., and the other 5 ft. Required the area of the trapezium.

13. A farm lying between two parallel roads 80 rods apart extends along one of these roads 120 rods, and along the other 320 rods. How many acres are there in this farm?

14. If a side of a regular decagon be 12 feet, and a line drawn from the middle of one side to the centre be 18.466 ft., what is its area? *Ans.* 1107.96.

15–22. A carpenter constructed eight regular polygons having respectively 3, 4, 5, 6, 7, 8, 9, and 10 sides. Each side was 3 feet in length. How many feet of lumber was required for each, allowing 20% for waste?

23, 24. The six sides of a plot, when measured, were found to be as follows: 32, 13, 15, 28, 17, and 16 rods; the first dia-

gonal measured 40 rods; the second, 48 rods; the third, 28 rods: how many square rods does the plot contain? How many ares?

25. The enclosing fence of a square field contains as many rails as there are acres in the field. The fence is 3 rails high, and each rail, exclusive of projecting ends, extends 11 feet. How many acres are there in the field.

ALGEBRAIC SOLUTION.

Let $x =$ number of acres or of rails.

$$\sqrt{x\left(160\times\frac{121}{4}\times 9\right)}\times\frac{4\times 3}{11}=x$$

As the removal of a radical sign squares the number covered,

$$x\left(\cancel{160}\times\frac{\cancel{121}}{\cancel{4}}\times 9\right)\times\frac{144}{\cancel{121}}=x^2$$

$$x = 360\times 144=51840.$$

436. *The Circle.*

DEFINITIONS.—A **circle** is a figure whose **circumference** or boundary is a curve line, every part of which is equally distant from the centre.

A **diameter** is a straight line passing through the centre and terminating at the circumference.

A **radius** is a straight line drawn from the centre to the circumference. All the radii of the same circle are equal.

The circumference is assumed to be divided into 360 equal parts, called **degrees** (360°); each degree is divided into 60 equal parts, called **minutes** (60′); and each minute into 60 equal parts, called **seconds** (60″). The abbreviations c, d, and r will sometimes be used to indicate respectively circumference, diameter, and radius. The length of a circumference whose d is 1 may be shown by the letter p. Thus $p \doteq 3.14159$.

XV. To find the circumference.

Take the product of the diameter into $3.14159=d\times p$.

XVI. The Diameter of a Circle equals

The circumference $\div$ 314159; or, $d = \frac{c}{p}$

XVII. To find the **Area of a Circle.**

Area $= \frac{c}{2} \times r$; or, $r^2 \times p$; or, $d^2 \times .7854$.

Note.—p is very nearly $3\frac{1}{7}$; $.7854 = p \div 4$.

Exercise CXXXVIII.

1–5. Required the areas of the five circles whose diameters are respectively 6 in., 6 ft., 12 yards, 15 rods, 4 miles.

6–10. What is the diameter of a circular acre; of a circle containing a square mile; of a circle equal to a square yard; to 60 square rods; to 50 acres?

11–20. What are the diameters corresponding to the circles represented as follows: 40 in.; 70 ft.; 38 yds.; 10 rods; 15 furlongs; 15 metres; 20 kilometres; 20 miles; 100 decimetres; 200 decametres?

21. A walk laid on the boundary of a circle is one-eighth of a mile long; what is the diameter of the circle?

22. How long is an arc of one-eighth of a circumference whose radius is 48 feet?

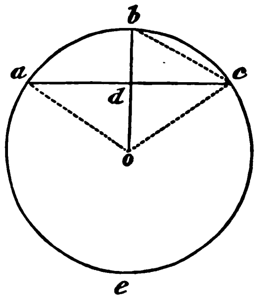
Definitions.

437. An **arc** is any part of a circumference.

A **chord** is the straight line which joins the extremities of an arc.

A **sector** is bounded by an arc, and the radii drawn to the extremities of the arc.

A **segment** is bounded by an arc and its chord.



a b c e is a circle, of which *o* is the centre; *o c*, *o b*, and *o a* are radii; *a c* is the chord of the arc *a b c*; *a b c o* is a sector; and *a b c*, or the part of the sector between the chord and the arc, is a segment; *d b* is called the **versed sine**: it is so much of the radius as lies between the chord and the arc.

438. XVIII. THE LENGTH OF ANY ARC EQUALS

Radius $\times p \times$ *number of deg. in the arc* $\div$ 180.

XIX. THE AREA OF A SECTOR EQUALS

Half the length of the arc $\times$ *r; or, it equals the product of the area of the whole circle multiplied by the number of degrees in the arc divided by* 360.

XX. THE AREA OF A SEGMENT EQUALS

The area of the sector minus the triangle of the chord and the radii. $\sqrt{2r \times d\,b} = b\,c$, or chord of half the arc *a c*.

THE AREA OF THE TRIANGLE EQUALS

The product of half the chord multiplied by the difference between the r. and the versed sine.

XXI. THE CHORD OF HALF AN ARC EQUALS

The square root of the product of the diameter and versed sine. $bc = \sqrt{d \times v.\,s.}$; or, $bc = \sqrt{2\,ob \times db}$.

XXI. *a*. TO FIND THE CHORD OF AN ARC WHEN THE DIAMETER AND VERSED SINE ARE GIVEN.

First, find the chord of half the arc. From the square of the chord of half the arc subtract the square of the versed sine and take twice the square root of the remainder.

XXI. *b.* THE LENGTH OF AN ARC MAY BE FOUND THUS:

First find the chord of half the arc, and then proceed as follows: From 8 *times the chord of half the arc subtract the chord of the arc, and divide the remainder by* 3.

EXERCISE CXXXIX.

1. A horse tied to a stake can graze an acre; how long is his halter?

2–8. In the figure *a b c o*, Art. 437, if the radius, or half-diameter, equals 10 inches, and *d b*, or the versed sine, equals 6 inches, what is the length of the chord *b c?* What is the length of the chord *a c?* What is the length of the arc *a c?* What is the area of the sector *a b c o?* What is the area of the segment *a b c?* What is the area of the triangle *a c o?* What is the area of the triangle *b c o?*

9, 10. What is the side of a square which will equal a circle whose r. is 60 ft.? Of a square which equals a circle whose circum. is 960 yards?

11–18. What is the side of the square inscribed in a circle whose diameter is 10 inches? 20 inches? Whose circumference is 40 ft.? 60 yds.? Whose area is 100 sq. ft.? 10 sq. rods? 1 sq. mile? 3 sq. metres?

19, 20. What is the diameter of an inscribed circle when the side of the square is 45 ft.? 72 ft.?

21–23. What is the circum. of an inscribed circle when the side of the square is 32 ft.? 75 ft.? 32 inches?

24, 25. Required the diameter of a circle equal to a square whose side is 50 ft.? Whose area is 32 sq. yds.?

26–28. Required the circumference of a circle equal to a square of 36 sq. yds.? 2 sq. miles? 25 sq. inches?

29–32. Find the areas of the circles whose diameters are as follows: 12 ft.; 12 rods; 100 yds.; 28 inches. (See **436.**)

33–35. What is the length of an arc of 36° whose radius is 12 inches; 42 inches; 3 metres?

36. If the radius of a certain sector be 7 ft., and the length of its arc be 4 ft., what is its area?

37. From a circle containing 480 square inches a sector was cut whose arc was 28°; what remained?

38. The radius of a circle is 18 inches; the versed sine of a segment of this circle is 4 inches; the chord of the arc is about 22 inches: what is the area of the segment?

Steps.—1st. Find the chord of half the arc. 2d. Find the length of the arc. 3d. Find the area of the sector. 4th. Find the area of the triangle formed by the chord and the radii. 5th. Subtract the area of the triangle from the area of the sector. (2d by Rule XXI. *b*.)

Results as follows: (1) 12 in. (2) 24.7 in. (3) 9×24.7 or 222.3. (4) $22 \times \frac{18-4}{2}$, or 154 in. (5) $222.3 - 154 = 68.3$ in.

39. What is the area of a segment if the radius is 10 in. and the versed sine is 5 inches (ch. of $\frac{1}{2}$ arc $= 10$; ch. of arc $= 17.32$).

40, 41. Req. area of a segment if the radius is 12 ft., and the v. s. is 3 ft.; of another whose radius is 15 yds. and versed sine 6 ft.?

42–46. What is the side of a square inscribed in a circle having a r. of 13 in.; in a circle having a circum. of 42 in.; a circum. of 85 ft.; a diam. of 24 yds.; an area of 47 sq. rods? (See Rule XXIII.)

Steps of Solution of Prob. 46. I. Find diam.

$$\sqrt{47 \text{ sq. rods} \div .7854} = \text{diam.}$$

II. Apply the rule. Thus: $47 \times .9003 \div$ diam. equal side of inscribed square.

47. How long is the arc *a b c*, if *o b* is 20 inches, and the angle *a o c* equals 85°? (See *figure* in Article 437.)

48. What is the area of the sector *a b c o* with the last conditions?

49–53. What is the area of a sector of 48°, if *o b* equals 30 inches; if *o b* is 42 inches; 23 feet; 5 metres; 2 miles?

Prove by the principle, that *all surfaces are in proportion as the squares of their like dimensions.*

439. *The Square.*

XXII. The Side of a Square which is equivalent to a given Circle equals $2 \times r \times .8862$; or, to the circum. $\times .2821$.

XXIII. *a.* The Side of an inscribed Square equals $2 \times r \times .7071$; or, circum. $\times .2251$; or, area $\times .9003 \div$ diam.

XXIII. *b.* The Area of an inscribed Square equals area of the circumscribed square divided by 2.

XXIV. The Square and Circle compared.

A side $\times$	1.1442	= Diameter of its circumscribing circle.				
" " "	4.443	= Circumference			"	"
" " "	1.128	= Diameter of an equal			"	"
" " "	3.545	= Circum.	"	"	"	"

For other methods of ascertaining the area of a sector or of a segment when there is given the chord of half the arc, together with the chord of the arc; or the chord of the arc and the versed sine; or when the diam. and the versed sine are given, see *Haswell*, p. 254.

440. *The Sphere.*

Definition.—A figure whose surface is at a uniform distance from its centre.

XXV. The **Surface of a Sphere** equals

The diameter $\times$ the circumference.

XXVI. The **convex Surface of a Segment** equals

The height $\times$ circumference of the sphere.

441. A **spheroid** is a figure formed by revolving a half-ellipse about one of its diameters. Spheroids are **oblate** (that is, flattened spheres) or **prolate** (or lengthened spheres). The *shorter diameter* is called the **conjugate**; the longer, the **transverse** diameter.

XXVII. THE SURFACE OF A PROLATE SPHEROID IS FOUND BY THE FOLLOWING

RULE.—*Square the diameters, multiply the square root of the half-sum of these squares by* 3.1416, *and multiply this product by the conjugate diameter.*

EXAMPLE : The diameters of a prolate spheroid are 20 and 28 inches ; what is the surface ?

$$\left(\frac{20^2+28^2}{2}\right)^{\frac{1}{2}}\times 3.1416\times 20=1528.074 \text{ sq. in.}$$

If the above dimensions had represented an **oblate spheroid** instead of the last factor, 20, the multiplier would have been the length of the *transverse* diameter, or 28—making 2139.3037.

XXVIII. THE SURFACE OF A CYLINDER EQUALS

The circumference × *length, added to the areas of the two circular ends.*

442. *Prisms.*

DEFINITION.—Figures whose sides are parallelograms, and whose ends are planes equal, similar, and parallel.

Triangular prisms have triangular ends, square prisms have square ends, etc.

The surface of a right prism equals the areas of its sides plus the areas of its two ends.

EXAMPLE.—A triangular prism, whose sides are equal, is 24 inches long, and each side is 12 inches wide. The area of the 3 sides is 864 in., of each end 62.3 in., total area 988.6 sq. in. (See Rule III.)

443. *Cylindrical Rings.*

DEFINITION.—Rings formed by the curvature of cylinders.

XXIX. TO COMPUTE THE SURFACE OF A RING,

Add together the inner diameter and the thickness of the ring, and multiply the sum by 9.6896 *times the thickness.*

Example.—A ring 1 inch thick whose inner diameter is 9 inches will have what surface? $(1+9)\times 1\times 9.6896=96.896$ sq. in.

444. *Pyramids.*

Definition.—Pyramids are figures whose bases have each three or more sides, and whose sides are plain triangles.

445. *Cones.*

Definition.—I. A cone is a figure described by revolving a right-angled triangle about one of its legs. Or,

II. It is a pyramid with an infinite number of sides.

XXX. The Surface of a Cone equals

The slant height $\times \frac{1}{2}$ *circumference of base, plus area of the base.*

Exercise CXL.

1–5. What is the surface of a ball whose diameter is 10 ft.? $(10\times 31.416.)$ What is the surface of a sphere whose circum. is 3 yds.? 3 ft.? 120 inches? 45 inches?

6. From a sphere 20 inches in diameter a segment 6 inches high was sawn: what was the convex surface of this portion? The circum. of a sphere 20 in. in diam. equals 62.832 inches. 62.832 in. $\times 6=376.992$ sq. in. $=$ convex surf. of the segment.

7. What is the area of the base of the segment, probl. 6.?

Solution: Rules XXI., XXI. *a*, etc. $\sqrt{(20\times 6)}=36\times 2=$ 18.33. 18.33, or diameter of base of segment, squared and $\times$ $.7854=263.91=$ area of base.

8–10. What is the convex surface of these segments of spheres: Diameter 15 in., height (or versed sine) 4 in.; diam. 20 ft., height 3 ft.; diam. 45 ft., height 10 ft.?

11–13. What are surfaces of the several bases of the above?

14–16. Required the surface of a prolate spheroid whose longest diam. is 40 in., and whose shortest diam. is 30 in. Also, the surface of another prolate spheroid whose diameters are 13 in. and 10 in.; of another, the diam. being 41 in. and 35 in.

17–19. Required the surface of three oblate spheroids having diameters equal to those mentioned in examples 14–16.

20–22. Required the surface of a cylinder 6 ft. long whose diameter is 3 ft.; 8 ft. long, diam. 1 ft.; 14 in. long, diam. 30 in.

23. The sides of a triangular prism are each 18 ft. long, and each side is 20 in. wide; what is the total area of its sides and ends?

24. What is the total area of the sides and ends of a triangular prism whose length is 40 inches if one of its sides be 20 in. wide, another 9 in. wide, and the other 16 in. wide?

25. What is the surface of a ring 2 in. thick if the inner diameter be 12 inches?

26–28. What is the surface of a ring 3 inches thick whose inner diameter is 3 feet? 24 inches? 10 inches?

29. What is the surface of a cone whose slant height is 12 feet, if its circumference be 12 feet?

30, 31. The height of a cone is 10 feet, its diameter is 10 feet. Required its slant height; required its surface.

Steps.—I. Take the square root of the sum of the squares of the height and radius for slant height.

II. Find the convex surface.

III. Find the surface of the base.

IV. Add these two areas.

32. The diameter of a flower-bed is 11.72 feet; the path about it is 44 in. wide; what is the area of the path?

33. The diameter of a cone is 9 inches; its perpendicular height is 20 inches: what is its convex surface?

34. If the surface of a certain sphere equals 100 sq. in., what is the surface of a sphere of which the radius is 23 times that of the first?

SOLIDS.

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446. *Parallelopipeds.*

DEFINITION.—A **Parallelopiped** is a figure having six quadrilateral sides, the opposite sides being parallel. The space occupied equals its volume. A **Cube** has six equal sides.

XXXI. THE VOLUME OF A CUBE OR OTHER PARALLELOPIPED EQUALS

The product of its length, width, and height.

Prisms.

DEFINITION.—Solids whose *ends* are planes equal, similar, and parallel, and whose *sides* are parallelograms.

XXXII. THE VOLUME OF A PRISM EQUALS

The area of the base $\times$ *by the height.*

447. XXXIII. THE VOLUME OF PRISMOIDS IS COMPUTED BY THE FOLLOWING:

RULE.—*To the sum of the areas of the two ends add four times the middle section and multiply the sum by one-sixth of the perpendicular height.*

NOTE.—The length and breadth of the middle section are respectively equal to half the sum of the lengths and breadths of the two ends. If the prismoid be very irregular, the sum of the areas of several cross-sections should be multiplied by the length, and the product be divided by the number of sections.

Polyhedrons.

448. A polyhedron is a regular solid lying within a certain number of equal and regular polygons. There cannot be

formed more than **five regular solids.** These five polyhedrons are: the **Pyramid** with four triangular faces, the **Cube** with six square faces, the **Octahedron** with eight triangular faces, the **Dodecahedron** with twelve pentagonal faces, and the **Icosahedron** with twenty triangular faces.

XXXIV. THE **Surface of a Regular Solid** EQUALS

The area of one face multiplied by the number of faces.

XXXV. THE **Volume of a Regular Solid** EQUALS

The square root of the cube of the entire surface, multiplied by .0517 *for a Pyramid of four equal sides; by* .06804, *for a Cube; by* .07311, *for an Octahedron; by* .08169, *for a Dodecahedron; by* .0856, *for an Icosahedron.*

NOTE.—When an edge or the radius of an ins. sphere or of a circ. sphere is given, see tables of *Haswell*, p. 272.

449. *Cylinders.*

XXXVI. THE VOLUME OF A CYLINDER EQUALS

The area of the base multiplied by the height.

XXXVII. THE VOLUME OF A **Cone** EQUALS

The area of the base $\times \frac{1}{3}$ *of the height.*

XXXVIII. THE VOLUME OF A **Pyramid** EQUALS

The area of the base $\times \frac{1}{3}$ *of the height.*

450. XXXIX. THE VOLUME OF A **Sphere** EQUALS

The cube of the diameter $\times$ .5236.

XL. *a*. THE VOLUME OF A **Segment of a Sphere.**

RULE.—*From three times the diameter of the sphere subtract twice the height of the segment; multiply this remainder by the square of the height, and the product by* .5236.

RULE XL. *b*. *To three times the square of the radius of its base add the square of its height; multiply this sum by the height, and the product by* .5236. $(3r^2+h^2)\times h\times .5236.$

451. XLI. TO COMPUTE THE VOLUME OF A RING.

To the inner diameter of the ring add the diameter of the metal; the sum $\times$ *the square of the diameter of the metal* $\times 2.4674$ *will equal the volume.*

TO COMPUTE THE VOLUME OF A **Cistern or Tub.**

RULE XLII. *Multiply the mean depth in inches by the mean diameter in inches, and this product by* .0034 *to find the number of wine-gallons.* (*See Appendix.*)

452. XLIII. THE CAPACITY OF A **Cask** EQUALS

The product of mean diameter $\times$ *length* $\times$ .0034.

Similar solids are proportioned to each other as the cubes of their like dimensions.

XLIV. TO FIND THE NUMBER OF FEET OF SQUARE-EDGED BOARDS IN A ROUND LOG 16 FEET LONG.

Square the diameter in inches less 4. $(D-4)^2$. *The yield of longer or shorter logs is in proportion to their length.*

EXERCISE CXLI.

1–6. What is the volume of a parallelopiped whose height is 8 in., width 7 in., and thickness 6 in.? What would it weigh if it were pure silver? Iron? Gold? Granite? Brick?

7–10. How long is that parallelopiped whose volume is 5832 cu. ft., if it be twice as long as it is wide, and twice as wide as it is thick? 12000 cu. in.? 64000? 64000 metres?

11. The area of a section of a regular prism is 35 feet, its length is 40 feet; what is its volume?

12. How many cubic yards in an embankment 100 yards long, the average area of 10 sections being 160 yds?

13. How many cubic in. in a tapering block 7 ft. long, one end measuring 6 in. by 8 in., the other, 16 in. by 18 in., the middle section, 11 in. by 13 in.?

14. A sculptor purchased a block of marble at $1.25 a cu. ft. The widths at each successive foot of its length were as fol-

lows: 4 ft., 3 ft. 10 in., 3 ft. 8 in., 3 ft. 10 in., 3 ft. 6 in., 3 ft. 4 in., and 3 ft.; in thickness it measured 3 ft. 4 in., 3 ft. 3 in., 3 ft. 1 in., 3 ft., 2 ft. 10 in., 2 ft. 9 in., and 2 ft. 7 in.; its length was 6 ft.: what did he pay for it?

15. What is the volume of a pyramid of four equal sides whose slant height is 2 ft., and whose width is 3 ft.?

Steps.—I. Area of one side $\times$ 4.

II. Area of base $+$ that of the 4 sides $=$A.

III. $\sqrt{A^3}\times.0517$. (*Vide* Rule XXXV.)

16. One edge of an octahedron is 7 in.; what is its volume?

Steps.—I. By Rule XIII., one side $= 7 \times .433$ sq. in.

II. Area of eight sides $= 24.25$ sq. in.

III. $\sqrt{24.25^3} \times .07311 =$ Answer.

17–19. What is the volume of a regular pyramid of four sides each of which is 3 ft. in slant height and 2 ft. in width? Of one whose base is 12 ft. in circumference, and whose slant height is 5 ft? Of one 8 ft. in perimeter or circumference, and 6 ft. in perpendicular height.

NOTE.—The area of the base, or 2×2 sq. ft., multiplied by $\frac{1}{3}$ of the perpendicular height equals the volume.

Ans. 8 cu. ft.

20. A cylinder 5 ft. long is 16 inches in circumference: how many cu. ft. does it contain?

21. The inside diameter of a cylindrical can is 12 inches. It holds 12 wine-gallons: how deep is it?

22. If a cylindrical can which holds 78.54 wine-gallons is as deep as it is wide, how deep is it?

$$\textit{Operation:}\ \sqrt[3]{\frac{78.54\times231}{.7854}} = 28.48.$$

23, 24. How deep is a similar cylinder whose capacity is 10 bushels? 15 litres?

25. A certain cone is 10 ft. high. Its base covers a circle 10 ft. in diameter. Required its contents.

$$\textit{Operation:}\ 10^2\times.7854\times(10\div3)=261.8.$$

26. If the base of a cone be 12 ft. in diameter, and its height be 18 ft., what is its volume?

27, 28. Estimating the specific gravity of iron at 7.8, and a cu. ft. of water at $62\frac{1}{2}$ lbs., what would be the weight of a sharp, hollow cone 5 feet in height, and 15 in. in diameter at the base, the shell being $2\frac{1}{2}$ in. thick? If it were filled with water, what would the water weigh? First answer: 701.65 lbs.

29–33. Required the volumes of a sphere whose diam. is 12 in.; whose diam. is 3 ft.; 20 ft.; 2.5 metres; 2 miles.

> NOTE.—A sphere may be regarded as if formed of many cones placed point to point. The sum of their bases equals the surface of the sphere. One-sixth of the diameter of the sphere equals one-third of the height of each cone. $3.1416 \div 6 = .5236$. A sphere 10 in. in diam. $= 10^3 \times .5236 = 523.6$ cu. in.

34. If a sphere 10 in. in diameter contain 523.6 cu. in., what is the diameter of a sphere whose volume is 20000 cu. in.? $523.6 : 20000 :: 10^3 : x^3 = \sqrt[3]{38197.}+$

> NOTE.—Solids are in proportion as the cubes of their like dimensions. (Note to probl. 47, *infra.*)

35. If a sphere whose volume is 20000 cu. ft. is 33.96 ft. in diameter, what is the diameter of a sphere whose volume is 7290 cu. ft.?

36. Required to divide transversely and equally a log 10 ft. long, 12 in. in diameter at the larger end, and 8 in. in diameter at the smaller end. The larger part $=$ 71.5496 in.
The shorter part $=$ 48.4504 in.

This problem may be solved thus:

Steps.—I. Calculate the volume of the cone formed by extending the log to a point$=$T. (T$=$completed cone.)

II. Calculate the volume of the 20-ft. cone added$=$P.

III. P taken from T will give the volume of the log, L.

IV. $P+\frac{1}{2}$ of L equals a cone which includes one-half of the log. Call this M.

V. $T : M :: 30^3 : x^3$. (Note to problem 33, *supra.*)

VI. Find x or length of the last cone M, from x^3, by extracting its cube root.

NOTE.—The letters T, P, and M are used merely for *convenience* to represent the number of cubic inches in the several cones. Make a drawing of the log extended.

37. A segment is cut from a sphere whose diameter is 100 inches; from the centre to the nearest point of the cut surface is 30 inches. How many cubic inches in the segment? (Rule XL.) Height of segment 20 in.

Steps.—I. $(100\times3)-(20\times2)=260$;

II. $260\times20^2\times.5236=54454.4$.

38. A segment 6 inches high is cut from a sphere whose circumference is 125.664 inches. What is the volume of the segment?

39. What is the volume of the segment of a sphere, if the radius of its base is 6 inches, and its height is 5 inches?

Rule XL. *b.* $(3\times36+25)\times5\times.5236$.

40. If the upper part of a certain balloon is 20 ft. in radius and 12 ft. high, how many cu. ft. of gas does this segment contain?

41. If from a sphere 50 feet in diameter a segment 5 ft. high be taken, what will remain?

42. Required contents of a cask 40 inches long, head diameter 26 inches, bung diameter 30 inches.

43. How many wine-gallons in a tun 6 ft. long, and 6 ft. in its greatest diameter, the head diameter being 66 inches?

44, 45. A miner casting his silver, found that it filled a mould 25 inches long, 8 in. wide, and 4 inches deep. A cu. in. of cast silver weighs .3788 lbs. avoir. His gold, when cast, filled a mould half as long as that for the silver, two-thirds as wide, and six inches deep. A cubic inch of gold weighs .6965 lbs. avoir. $371\frac{1}{4}$ grains of silver are worth a dollar. If pure gold be worth 16 times as much as pure silver, what was each of the two ingots worth?

46. A sphere equals .5236 of a cube of the same diameter. Required the value of a sphere of pure silver, the diameter being 2 inches.

47. How many such spheres will 342730.957 lbs. of silver make ?

NOTE.—All *similar* solids are in proportion as the cubes of their like dimensions. If the diameter of one sphere be three times as great as that of another, the volume of the first sphere will equal twenty-seven times that of the second. So a fish which is shaped exactly like another fish 4 times as long is but one sixty-fourth as heavy as that other.

TO COMPUTE THE VOLUME OF AN IRREGULAR BODY.

RULE.—*Weigh it by the usual method, and also in fresh water. Divide the difference of these weights by* 62.5, *and the quotient will express the volume in cu. ft.*

EXERCISE CXLII.

1, 2. A mass of Quincy granite whose weight is 828.75 lbs. when weighed in water was found to weigh 516.25 lbs.; what is its volume ? What does a cu. ft. of granite weigh ?

3, 4. A cannon, when weighed in water, appeared to lose 593.75 lbs. How many cu. ft. of gun-metal did it contain, and what was its weight, gun-metal weighing 466.5 lbs. to the cu. ft. ?

5–8. A block of limestone lost, when weighed in sea-water, 1600 lbs. Sea-water weighs 64 lbs. to the cu. ft.; limestone weighs 197 lbs. to the cu. ft. What was the weight of the block in question ? What was its volume, its specific gravity being 2.386 ? What would a block of granite of the same volume weigh ? What a mass of cast-iron of the same size ?

ARITHMETICAL PROGRESSION.

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453. *A series of numbers* increasing or decreasing by a constant *difference* is an ***Arithmetical Progression,*** as, 2, 5, 8, 11, 14, 17, 20; or, $\frac{7}{8}$, $\frac{6}{8}$, $\frac{4}{8}$, $\frac{3}{8}$. The numbers are called **terms,** the first and last the **extremes,** the others the **means.** In the first example the first term or a is 2; the last term, l, 20; the common difference, d, 3; the number of terms, n, 7; and the sum of all the terms, s, equals 77.

The common *formula* for the last term is deduced thus: The 2d term equals the difference plus the 1st term, or $a+d$; the 3d term equals the 2d term$+d$, or $a+2\times d$; the 4th term equals the 3d term $+\ d$, or $a+3\times d$; etc. Hence the 19th term $=a+18\times d$, and $l=a+(n-1)\times d$; and in general the **last term** *will equal the first term plus the product of the common difference into the number of terms less one.*

A body falls 16 feet in the first second of its descent, 48 feet in the second second, 80 feet in the third second, etc.; how far will it fall during the tenth second? $16+(32\times 9)=$ **304.**

$$
\begin{array}{ll}
\text{Series.} & 2+5+8+11+14+17+20=\text{sum.} \\
\text{Transposed.} & \underline{20+17+14+11+8+5+2=\text{sum.}} \\
 & 22+22+22+22+22+22+22=\text{sum}\times 2=
\end{array}
$$

$n\times(l+a)$ or $7\times(20+2)$. And $s=\frac{n}{2}\times(a+l)$.

Hence, **the sum** *equals half the number of terms multiplied into the sum of the extremes.*

Example. What is the sum of 1, 2, 3, 4, etc., up to and including 200? $(1+200)\times(200\div 2)$.

The number of terms *will equal the difference of the extremes divided by the common difference plus* 1. $n = \left(\frac{l-a}{d}\right) + 1.$

In the example first given, the difference between the first and last terms is 18 ; this, divided by the common difference, 3,=6 ; add 1 to this and the sum, 7, equals the number of terms. $(20-2) \div 3 + 1 = 7.$

The common difference *equals the quotient obtained by dividing the difference of the extremes by the number of terms less* 1 $(l-a) \div (n-1)$.

Example. The difference of the extremes is 24, the number of terms is 13; then the common difference must be 24÷12, or 2.

Exercise CXLIII.

1, 2. If a triangle be so paved that, commencing with 1 brick, the 88 rows constantly increase by 2 bricks, how many bricks in the last row ; and how many in all ?

3, 4. 12 persons contributing to a charitable object, the 1st gave $1 ; the 2d gave 95 cents, the 3d, 90 cents ; and so on in a descending series ; how much did the last one give ; and how much was received from these 12 men ?

5, 6. For digging a well the contractor was to receive 12 cents for the first foot, 15 cents for the second, and so on increasing according to the depth. He was paid $2.22 for digging the last foot : how deep was the well ; and how much did he get in all ?

7–9. If a man bring a basket of fruit from each of 20 trees standing in a line 20 feet apart, to a wagon stationed under the first tree, how far must he walk in all ? How far did he travel to and fro for the 12th basket ? For the 17th basket ?

10. What is the amount due for rent on a store worth $400 per year, nothing having been paid for 12 years ; simple interest on each year's rent ?

GEOMETRICAL PROGRESSION.

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454. A series of numbers which increases by a constant multiplier or which decreases by a constant divisor is called a **Geometrical Progression**; as, 1, 2, 4, 8, etc.; or, 3, 1, $\frac{1}{3}$, $\frac{1}{9}$; or, as, .3333.

455. The common multiplier or divisor is called the **ratio.** The first and last numbers are called the extremes. The abbreviations in common use are for the *ratio, r;* for the *first term, a;* for the *last term, l;* for the *number of terms, n;* for the *sum of all the terms, s.*

Three of these being given, the remaining two can be computed.

NOTE.—To find the number of terms, logarithms are required.

456. In order to comprehend the rules which follow, pupils should carefully observe the construction of several geometrical series. Let

I. $3+6+12+24+48$ be analyzed. It equals $3+3\times2+3\times2^2+3\times2^3+3\times2^4$.

The first term, or a, is 3; the ratio, r, is 2; the number of terms, n, is 5. The fifth or last term, l, is equal to 3×2^4, or $a\times r^{n-1}$. Hence $l=a\times r^{n-1}$ expressed without abbreviation, thus: The last term = the first term multiplied by that power of the ratio denoted by the number of terms less one.

457. We may deduce the value of s as follows:

I. $2+6+18+54+162=s$.

II. $6+18+54+162+486=3s$.

(The 1st series multiplied by 3 equals the 2d series.)

Subtracting the 1st series from the 2d, term by term, there remain $486-2=2s$.

It will be observed that 486 equals the last term of the 1st

series multiplied by r, and that $486-2=lr-a$. Since $2s=3s-s=rs-s$, or $(r-1)s$, it follows that $lr-a=(r-1)s$.

If each of these equals be divided by $(r-1)$, s will stand equal to $(lr-a)\div(r-1)$, or, as above, $s=\frac{lr-a}{r-1}$.

RULE.—*Multiply the last term by the ratio, from the product subtract the first term, and divide the remainder by the ratio less* 1.

EXERCISE CXLIV.

1, 2. How many ancestors hast thou, reckoning 20 generations back; and how many of the twentieth degree?

3. Allowing 5000 grains of wheat to a quart, how much wheat would equal the last year's crop produced from the five kernels found 25 years since in the wrappings of a mummy?

4. If $1 had been put at interest A.D. 1492, and had doubled in value every 12 years, what would it have amounted to A.D. 1877?

5. The 1st term of a descending series is 486, the ratio is 3, what is the 6th term?

NOTE.—In a decreasing or descending series the ratio may be regarded as a fraction whose numerator is 1.

6. A thrasher agreed to work 20 days for 4 grains of wheat the 1st day, 12 grains the 2d day, 36 grains the 3d day, etc.; what would his wages amount to, if 3840 grains of wheat make a pint, and a bushel were worth $1.25?

458. In a series continued to an infinite number of terms, constantly decreasing, the last term, or l, $=0$. In the formula $s=\frac{lr-a}{r-1}$, $lr=0\times r$ or 0, hence $s=\frac{-a}{r-1}$, or $\frac{a}{1-r}$.

EXERCISE CXLV.

1. Required the sum of the infinite series $1+\frac{2}{3}+\frac{4}{9}+\frac{8}{27}$, etc.
2. What is the sum of the infinite series, $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$, etc.?

3. Required the sum of the repetend .333+.

4. Required the sum of $8+1+\frac{1}{8}+\frac{1}{64}$, etc.; continued to infinity.

5. A father gave his daughter $100 on her wedding day, and doubled this sum for the next 9 years; what was his daughter's dowry?

REPETENDS.

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459. In order to reduce a proper fraction to a decimal, it is necessary to multiply the numerator by 10 or by some power of 10. If the fraction has been reduced to its lowest terms, and its denominator contain *any other factor than* 2 *or* 5, its value *cannot be expressed accurately as a decimal;* for the multiplying of its denominator by 10 will not make a product which is a multiple of the denominator. If the reduction be attempted, the same quotient figure or figures will be repeated again and again without end. Thus: Let it be required to reduce $\frac{1}{3}$ to a decimal form. Result .333, etc. Reduce $\frac{1}{6}$ to a decimal form. Result .166, etc. Such results are called ***Repetends.***

460. Repetends are, in fact, **infinite decimals,** and as such come under the head, INFINITE SERIES in *Geometrical Progression*. .3333, etc., is expressed thus, $.\dot{3}$; .1666, etc., is expressed thus, $1\dot{6}$; and .020202, etc., would be expressed thus, $.\dot{0}\dot{2}$.

461. If a single figure repeats, it is written *once with a point above it;* if several figures taken together repeat, a point is placed over the *first and the last of the set.*

462. The figure or set of figures which repeat is called a **period.** Thus: 241 in $241\dot{2}4\dot{1}$ is a period.

463. A **pure repetend** is one in which all the figures constantly repeat.

464. A **mixed repetend** is one in which one or more figures at the left do not repeat.

$$\tfrac{1}{9}=.\dot{1}\,;\ \tfrac{1}{99}=.\dot{0}\dot{1}\,;\ \tfrac{1}{999}=.\dot{0}0\dot{1}\,;\ \tfrac{1}{9999}=.\dot{0}00\dot{1}.$$
$$\tfrac{7}{9}=.\dot{7}\,;\ \tfrac{27}{99}=.\dot{2}\dot{7}\,;\ \tfrac{504}{999}=.\dot{5}0\dot{4}\,;\ \tfrac{2007}{9999}=.\dot{2}00\dot{7}.$$

465. It will appear from the inspection of these examples that a pure repetend of one figure must be derived from a common fraction whose denominator is 9; a repetend of two figures arises from a fraction having 99 for a denominator; a repetend of three figures, 999; of four figures, 9999; etc., etc.

Mixed repetends are derived from such fractions as $\frac{1}{6}$ and $\frac{1}{15}$.

$$\tfrac{1}{6}=.1666\text{, etc., or }.1\dot{6}\,;\ \tfrac{1}{15}=.0666+\text{, or }.0\dot{6}.$$

466. *The figures which do not repeat are,* in fact, *decimal figures.* In ascertaining the precise value of such fractions as $.1\dot{6}$, the two parts must be considered separately. Thus $.1\dot{6}=\frac{1}{10}+\frac{6}{90}$, or $\frac{15}{90}$, or $\frac{1}{6}=.1\dot{6}$.

Let it be required to reduce $.12\dot{3}\dot{4}$ to a simple fraction.

OPERATION:

$$\tfrac{12}{100}+\tfrac{34}{9900}=\tfrac{1188}{9900}+\tfrac{34}{9900}=\tfrac{1222}{9900}=.12\dot{3}\dot{4}.$$

To multiply a number by 99 is to take it once less than a hundred times. Thus: $12\times99=1200-12=1188$. $1200+34=1234$, and $1200-12+34=1234-12$.

In general, the mixed repetend diminished by the terminate part will equal the sum of the numerators of the separate parts when reduced to a common denominator.

As the reduction of the two fractions required for the expression of a mixed repetend necessitates the multiplication of both terms of the first fraction by 9, 99, or 999, etc., it is obvious that

467. A Mixed Repetend may be reduced to a Common Fraction thus:

Rule.—*For the numerator, take the whole repetend less the decimal part. For the denominator, write as many nines as there are figures in each period, and to these nines suffix as many ciphers as there are decimal figures.*

Thus: $.12\dot{3}\dot{4} = \frac{1234-12}{9900}$, or $\frac{1222}{9900}$.

468. To express the exact equivalent of a Pure Repetend.

Rule.—*Write the given quantity for a numerator, and for a denominator write as many nines as there are repeating figures.*

469. Repetends whose denominators are equal are **similar.**

470. One or more periods of a pure repetend may be written as a decimal part, and the period may thus be made to begin further toward the right without altering the value of the expression.

471. Two or more periods of any repetend may be taken together as one period.

Thus: $.\dot{2} = .2\dot{2} = 22\dot{2}\dot{2} = 22\dot{2}2\dot{2}$.

Required to add $.01\dot{6}$ $.0\dot{1}\dot{6}$ $.\dot{0}1\dot{6}$.

$$.016\dot{6}6666\dot{6} = \tfrac{16666650}{999999000} = \tfrac{15}{900} \text{ and } .01\dot{6} = \tfrac{15}{900}$$
$$.016\dot{1}6161\dot{6} = \tfrac{16161600}{999999000} = \tfrac{16}{990} \text{ and } .0\dot{1}\dot{6} = \tfrac{16}{990}$$
$$.016\dot{0}1601\dot{6} = \tfrac{16016000}{999999000} = \tfrac{16}{999} \text{ and } .\dot{0}1\dot{6} = \tfrac{16}{999}$$
$$\overline{.048844298}$$

472. That **the periods may not be broken,** the grand period must in this case have 6 figures, 6 being the least common multiple of 1, 2, and 3, the number of figures in the periods of the respective addends.

473. For a similar reason, the addends must have 3 figures in the decimal part, as **the grand period of each addend must begin at the same order with that of the others.**

474. TO ADD OR SUBTRACT REPETENDS.

RULE.—*If they are unlike, make them similar. Find their sum or difference, as in whole numbers. Point off the results like the addends, or like the subtrahends.*

475. TO MULTIPLY OR DIVIDE REPETENDS.

RULE.—*Reduce them to common fractions, by Art.* 467, *and seek the required results by the rules for common fractions.*

476. Many practical operations require that decimal figures shall be extended to six or more places. This is the case in tables for engineers, surveyors, and architects. A knowledge of the nature of repetends is often productive of a great saving of time.

For ordinary purposes, a repetend, if extended to four places, *may be treated as a decimal.*

EXERCISE CXLVI.

Reduce each of the following common fractions to the form of a decimal or of a repetend:

1–20. $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{5}$; $\frac{2}{7}$; $\frac{1}{9}$; $\frac{7}{99}$; $\frac{10}{21}$; $\frac{1}{6}$; $\frac{5}{66}$; $\frac{12}{29}$; $\frac{4}{51}$; $\frac{3}{81}$; $\frac{3}{8}$; $\frac{23}{100}$; $\frac{4}{10}$; $\frac{4}{11}$; $\frac{4}{13}$; $\frac{4}{17}$; $\frac{101}{100}$; $\frac{11}{27}$.

EXERCISE CXLVII. (See also SUB-EXERCISE.)

Express the following as common fractions:

1–5. $.\dot{8}$; $2.\dot{7}5\dot{2}$; $2.\dot{0}\dot{7}$; $2.0\dot{7}$; $.\dot{2}0\dot{7}$.

6–10. $5.\dot{8}9\dot{6}$; $.\dot{1}00\dot{7}$; $.1\dot{0}0\dot{7}$; $.\dot{1}008\dot{0}$; $.000\dot{1}\dot{7}$.

11–15. $.5\dot{9}\dot{1}$; $.59\dot{1}$; $.\dot{5}9\dot{1}$; $.591\dot{0}$; $5.\dot{9}1\dot{0}$.

SUB-EXERCISE.

1–3. **Add** each group of the above (5 in a group).

4, 5. **Subtract** the 12th from the 11th; the 9th from 5th.

6–8. **Multiply** each of the first five by 1.05; by $.\dot{5}$; by $2\frac{1}{6}$.

9, 10. **Divide** each one of the 2d group by 3; by $1.\dot{3}$.

APPENDIX.

ARITHMETIC EXTENDED.

In order that the best disciplinary results may be secured, it is desirable that pupils should be practised in abstract reasoning. They should be taught to **generalize.** A knowledge of the simpler applications of letters and symbols tends greatly to abbreviate and elucidate mathematical operations. There is no sound reason why the "line fence" between Arithmetic and Algebra should stand where it does. Why should not a pupil be encouraged to formulate his rules and statements within the briefest compass? By the natural device of initials for words, and by the use of signs, pupils may easily write out such expressions as $l \times b \times d$=volume; $i = p \times r \times t$; $\frac{D^3\pi}{6}$=vol. of a sphere, and state within a finger's breadth the conditions of a problem which, written out at length, would fill half a page. These simple matters should not be restricted to the wearers of academic gowns. The word "ALGEBRA" has too long been used as a term to conjure with. Its elementary processes should be made as familiar to the young as those of Arithmetic.

It is suggested that, in presenting **signs** and **letters**, teachers should proceed somewhat as follows. (The instances given are only examples. The instructor will, of course, in each case, add a considerable number of similar expressions.)

Addition. Sign, + (plus). $7+6$ means seven and six. Equals is shown by the sign =. $7+6=13$; or, $a+b=c$, a standing in this instance for 7, b for 6, and c for 13. $3+5=8$ or $a+b=c$. Here $a=3$, $b=5$, and $c=8$.

Subtraction. Sign, –. This sign, –, is called minus. Its use will appear from the following expressions: $20-7=13$; $10-3=7$; $a-b=c$, a standing for 20, b for 7, and c for 13, in the one instance; and a for 10, b for 3, and c for 7, in the other.

Multiplication. Sign, $\times$. $4\times5=20$; $3\times5\times2=30$. Letters, when written together, are to be regarded as factors. Thus: if $a=3$; $b=5$, and $c=2$, then $abc=30$. A parenthesis, ( ), or a vinculum, $\overline{}$, serves to so connect two or more quantities that they may stand as one whole $(14-5-2)\times3=21$ for $14-5-2=7$ and $7\times3=21$.

Division. Sign, $\div$, or : Thus, $12\div2=6$; $8:4=2$. The Dividend is sometimes written over the divisor thus $\frac{12}{2}=6$.

Involution. Sign, a small figure at the right shoulder of a number or letter. $4^3=4\times4\times4$. $a^4=a\,a\,a\,a$.

Evolution. Sign $\sqrt{}$ or $\sqrt[3]{}$ Thus, $\sqrt{25}=5$; $\sqrt[3]{27}=3$; $64^{\frac{1}{3}}=4$.

Any letter may be used to represent **any number**. A letter which stands for one number in one problem may stand for a different one in the next.

Such expressions as $4-1=3$, $2+5-1=6$, $3\times4=12$, $18\div3=6$, and $ab=x$ are called **equations.**

AN EQUATION is any formula which expresses **equality.** It consists of **two members;** the one on the left hand of the sign of equality is called the first member; that on the right hand is called the second member. Thus: In the equation $14=10+4$, 14 is the first member, and $\overline{10+4}$ is the second member. Either or both members may consist of several parts or terms. **A term** is an entire quantity, as 7, ¾, 5×6, $ab\,(a-b+c)$. Many of the expressions already employed in the last eight paragraphs are equations.

The operations on equations are based on simple truths called **axioms.** The following are those generally referred to:

1. If equals be added to equals the sums will be equal.
2. If equals be subtracted from equals the remainders will be equal.
3. If equals be multiplied by equals the products will be equal.
4. If equals be divided by equals the quotients will be equal.
5. Like powers of equals are equal.
6. Like roots of equals are equal.
7. Quantities equal to the same quantity are equal to each other.

(See Axiom VII., p. 18.)

When an **equation** has **fractional terms,** if every separate term be multiplied by the common denominator the terms will appear as integers. Axiom 3.

As $2+5=7$, and $3+8=11$ ∴ (therefore) $2+5+3+8=7+11$, Axiom 1; so if $a+b=c$, and $d+e=f$, then $a+b+d+e=c+f$.

As $10-7=3$, $10-7+4=3+4$, Axiom 1; so if $a-b=x$, then $a-b+c=x+c$. Again, if $x+4=16$, then $x=16-4$, Axiom 2.

As $8=4+3+1$, $5\times8=5\times(4+3+1)=(5\times4+5\times3+5\times1)=20+15+5=40$. Axiom VII. (See page 18.) So if $a=b+c+d$, then $ra=rb+rc+rd$.

As $7+5=5\times7$, so $ab=ba$.

If $a=b+c$, and if $d=r+s$, then $ad=br+bs+cr+cs$. Axiom VII. (See page 18.)

As $\frac{28}{2}=\frac{16+8+4}{2}$; so $\frac{a}{2}=\frac{b+c+d}{2}$, provided $a=b+c+d$. AXIOM VIII. (See page 21.)

If $36\div18=36\div(9\times2)$, then $\frac{36\div9}{2}=36\div18$.

It is customary when solving an **arithmetical problem** to proceed from **left to right**, taking each number in its turn, unless it is joined with another by a parenthesis or vinculum.

By Arithmetic $18-6\times3+24\div15=4$. In Algebra, two or more terms connected by $\times$ or $\div$ are considered as making one whole, or as if within a parenthesis. Thus: $18-6\times3+24\div15=18-(6\times3)+(24\div15)=18-18+1\frac{3}{5}=1\frac{3}{5}$.

Classes in arithmetic will, of course, use the arithmetical method.

When the conditions of a problem are set forth in due mathematical form the statement takes the form of an equation. It is customary to represent the answer required by writing it as the first member with one of the last letters of the alphabet. Thus: What is the interest on \$42 for 4 years at 6 per cent.?

Statement: $x=\$42\times.06\times4$.

If a box contain 420 cubic inches, its height being 3 inches, and its width 10 inches, what is its length?

Statement: $x=420\div(3\times10)$.

In actually working out the problem, the mental processes are the same as those employed in Arithmetical Analysis. (Art. 403.)

GENERAL RULE.—*Make like changes in both members, so operating that the unknown quantity or answer shall finally stand alone as one member of an equation opposite a known quantity as the other member.*

1. A boy bought a pear and a banana for 9 cents. The banana cost twice as much as the pear. What was the cost of each?

Let $x=$ the price of the pear, then $2x=$ the price of the banana. $\therefore$ $x+2x=9$. Since $3x=9$, $x=3$. (Axiom 4.) $\therefore$ the price of a pear is 3 cents; the price of a banana 6 cents.

2. If the greater of two numbers is 5 times the less, and their sum is 60, what are the numbers?

Let $x=$ the less, then $5x=$ the greater. $x+5x=60$, $6x=60$, and $x=10$. Hence the numbers are 10 and 50.

Question 1 *supra* might have been solved thus: By the conditions, 9 cents = the price of a pear + the price of a banana, or the price of 3 pears. Since three pears cost 9 cents, 1 pear will cost 3 cents, and the banana, which cost as much as 2 pears, cost 6 cents.

3. Interest is considered as a percentage of the principal, and is found by multiplying the principal by the product of the rate and time, *e.g.*, $i=p\times r\times t$. Since $i=p\times r\times t$ (*e.g.*, \$6=\$50 × 6 per cent. × 2); then

$$(1)\ \frac{i}{p\times r}=t;\quad (2)\ \frac{i}{p\times t}=r;\quad \text{and } (3)\ \frac{i}{r\times t}=p.$$

(1) $\frac{6}{50\times.06}=2$. Hence the time = 2 years.

4. If the volume of a sphere is 523.6 cu. inches, required its diameter.

Since the volume of a sphere $=\frac{D^3\pi}{6}$, $v\div\frac{\pi}{6}=D^3$.

$523.6\div.5236=1000=D^3$.

$\sqrt[3]{1000}=D=10$.

5, 6. The volume of a cylinder$=l\times D^2\times.7854$. If the contents of a log $=78.54$ cu. ft., and its diameter is 3 ft., what is its length? $l=78.54\div3^2\times.7854$. Then $l=100\div9=11\frac{1}{9}$ feet.

If the length had been stated as 12 feet, and the diameter had been required, then $D^2=78.54\div(12\times.7854)=8\frac{1}{3}$. Since $D^2=8\frac{1}{3}$, $D=\sqrt{8.\dot{3}33\dot{3}}$, or 2.887 ft. (See also Articles 453 and 454.)

MECHANICS.

The force of gravity causes the downward pressure of a body, or what is called its weight.

Bodies left unsupported fall to the earth.

TABLE OF THE LAWS OF **Falling Bodies.**

Times (in seconds),	1	2	3	4	5
Velocities (in feet) at close of times,	32	64	96	128	160
The space for each time (feet),	16	48	80	112	144
The whole spaces (feet),	16	64	144	256	400

THE **distance** TRAVERSED BY A FALLING BODY EQUALS *the square of the time multiplied by* 16.

THE **velocity** ACQUIRED BY A FALLING BODY AT THE END OF A GIVEN NUMBER OF SECONDS EQUALS *thirty-two feet multiplied by the number of seconds.*

The attractive force at the equator is 32.09 feet, and the centrifugal force is .111216 feet.

A mass weighing 1,000 lbs. at the equator would weigh 1,005 lbs. at the pole.

The velocity imparted to a pendulum of a given length is proportioned to the attraction of the earth. This attraction is greater at places near the centre than at places further removed. In high latitudes the surface is nearer the centre than it is in the vicinity of the equator. The polar radius is 13.246 miles less than the equatorial radius. To compensate the deficiency in attraction, the pendulums of clocks

within the tropics are shortened. The length of a pendulum beating seconds at the following places must be as follows:

Spitzbergen, lat.	79° 50′	39.216 in.	New York, lat.	40° 43′	39.102 in.
Greenland, "	74° 32′	39.204 "	C. of G. Hope, "	33° 56′	39.079 "
St. Petersburg, "	59° 56′	39.170 "	Rio Janeiro, "	22° 55′	39.045 "
Paris, "	48° 50′	39.130 "	Maranham, "	2° 32′	39.013 "

NOTE.—A metre, according to the U. S. Coast Survey, is 39.3685 inches.

THE **force** WHICH A MOVING BODY REPRESENTS EQUALS *its* **weight** *multiplied by its velocity* (**momentum**).

ITS **impact** EQUALS *its weight* × *square of its velocity.*

Theories relating to Machinery.

THE LEVER.—Let *fw* indicate the distance of the weight from the fulcrum, prop, or pivot, and *fp* the distance of the power from the fulcrum. Then, letting P stand for power, and W for weight,

$$P : W :: fw : fp.$$

WHEEL AND AXLE.—Let *rw* indicate the radius of the wheel, and *ra* the radius of the axle. Then

$$P : W :: ra : rw.$$

WHEELS AND PINIONS.—Let *p.l.* equal the number of leaves in all the pinions, and *w.t.* the number of teeth in all the wheels. Then

$$P : W :: p.l. : w.t.$$

PULLEYS.—W=P × 2 × number of movable pulleys.

INCLINED PLANE.—P : W : : height of plane : length of plane.

SCREW.—Let W indicate resistance or pressure, *d.t.* the distance between the threads, and *c.p.* the circumference or distance traversed by the power. Then

$$P : W :: dt : cp.$$

POWER.—This is usually estimated by comparison with **Horse power.** The unit of horse power equals the power required to lift 33,000 pounds avoirdupois one foot high in one minute.

In drawing loads on a level road, the strength of a **man** as compared with that of **other animals** is as follows: Man, 1; Horse, 7; Mule, 7; Ox, 4 to 7; Dog, .6.

A **man's power** employed to the *best* advantage is equal to the raising of 2,000,000 lbs. 1 foot per day; employed at a windlass, it is equal to 1,250,000 lbs. 1 foot high per day.

The power of coal at regular work is 100,000,000 lbs. 1 foot high to the bushel of 84 lbs., under the engine.

1. If the force necessary to move a R.R. train for a certain period at the rate of 25 miles an hour be estimated as 1, what will represent

the force necessary to move twice as heavy a train for the same length of time at the rate of 40 miles an hour? ($25^2 : 2 \times 40^2 : : 1 : x$.)

2. Which has the greater striking force or impact, a canal-boat weighing 300 tons moving 5 miles an hour, or a cannon-ball weighing 100 lbs. moving 1,200 feet in one second?

3. A weight of 300 lbs. is suspended 6 inches from the middle of a pole 5 feet long, the pole being supported at the ends. Required the bearing at each end?

4. If a pole 24 feet long, weighing 100 lbs., be so borne by two men, "Stout" and "Stripling," that 6 feet project beyond Stout, and 2 feet beyond Stripling, and if a weight of 100 lbs. be suspended from the pole 10 feet from Stout, how many pounds does he carry? How many pounds are borne by Stripling?

5. How far from the centre of a double whiffle-tree 4 feet in length must the draw-bolt be inserted, so that the stronger horse of a team must draw three-fifths of the load?

6. Two toothed wheels with shafts and pinions are in gear. There are 100 teeth in each wheel and 12 teeth in each pinion. If the first wheel be moved with a power of 80 lbs., with how much force will the last shaft be turned?

7. A wheel 10 feet in diameter has an axle 10 inches in diameter; what force exerted at the circumference will sustain 1,000 lbs. suspended from a rope 4 inches in diameter wound about the axle?

8. If 1 represents the force required to keep a train moving at a certain rate on a level, what will indicate the force required to move the same train at the same rate up a grade of 80 feet to the mile?

9. The threads of a screw are 1 inch apart. The power is applied 8 feet from the centre. What weight will it raise if the friction occasion a loss of 25 per cent.?

10. If a "Cunarder" steam 20 miles during one hour, 15 miles during the next hour, and 12 miles during the next hour, under the same conditions as respects wind and tide, what would be the comparative amounts of coal expended during each hour?

11. If the resistance of the air to the passage of a ball at a certain rate for a mile be 20 tons, what will be the resistance to the passage of a ball 8 times as heavy, moving twice as fast, over the same space?

12. To move a weight of 7200 lbs., 4 movable pulleys and a windlass are employed. The windlass crank is 24 inches long, the axis is 3 inches in diameter. The rope is 2 inches thick. Allowing 30 per cent. for friction, what power is required?

TENSILE STRENGTH OF MATERIALS.—This is proportional to the area of a transverse section. The *unit of area* is 1 square inch. The bar to be tested is secured at one end, and weights, gradually increased, are suspended from the other, until it is broken. In the following table the *greatest* absolute strength is expressed in *pounds*:

Steel, untempered,	127094	Zinc,	2820
" tempered,	153741	Lead,	887
" cast,	134256	Birch,	12225
Iron, bar,	84611	Elm,	15040
" wire,	113000	Larch,	12240
" cast,	19464	Oak,	25851
Silver, "	40997	Box,	24043
Copper, "	37380	Ash,	23455
Brass, "	19472	Pine,	14965
" "	58931	Fir,	12876
Gold,	65237	Hemp,	8746
Tin, cast,	4736	Cable, 5 in.,	4860

NOTE.—These estimates are, for the most part, *higher* than those usually given.

"The strength of cords is in proportion to the fineness of their strands and of the material. They are weakened by overtwisting. Damp cords are stronger than dry ones, twisted than spun, tarred than untarred, unbleached than bleached. Silk cords are three times stronger than flaxen cords."—*Silliman.*

TRANSVERSE STRENGTH OF MATERIALS.—The *transverse strength* of a substance is found by reducing it to the uniform measure of 1 inch square and 1 foot in length, and suspending the weight from one end, so as to operate in a direction at right angles to its length. The table shows the **value, for general use,** of the following materials. The *actual* breaking-weight is usually about four times the number of pounds here given :

Cast-Iron,	125 to 250	Oak, Eng.,	45
Steel,	170 to 350	Pine, pitch,	45
Wrought-Iron,	160 to 210	" white,	30
Copper,	55	Teak,	60
Brass,	58	Flagging,	10
Ash-wood,	55	Freestone,	6
Beach,	32	Granite,	7
Birch,	40	Slate, Bangor,	30
Chestnut,	53	Limestone,	$3\frac{1}{2}$
Oak, Am.,	50	Marble,	$1\frac{1}{2}$

Wood, when seasoned, is from 10 to 40 per cent. stronger than when green.

TO COMPUTE THE TRANSVERSE STRENGTH OF A RECTANGULAR BEAM OR BAR WHEN IT IS SUPPORTED AT BOTH ENDS AND THE WEIGHT IS SUPPORTED AT THE MIDDLE.

RULE.—*Multiply the value of the material by 6 times the breadth and the square of the depth in inches, and divide the product by the length in feet.*

A beam will support twice as much weight distributed over its entire length as if the weight were resting at the centre. A thin, broad beam resting on its edge is from 5 to 10 times as strong as when laid upon its side. In **round timbers** the strength is inversely as their length, and directly as the cubes of their diameters. A cylinder whose diameter is three times that of another cylinder will support 27 times as much.

The strongest rectangular beam which can be sawed from a cylinder is one whose breadth equals $\sqrt{\frac{1}{3}} \times$ diameter, and whose depth is $\sqrt{\frac{2}{3}} \times$ diameter. Thus, the strongest beam which can be sawed from a log 20 inches in diameter will be $.5774 \times 20$ inches in breadth, and $.8165 \times 20$ inches in width.

Of all beams, hollow rectangular iron beams are strongest in proportion. Their height should be greater than their breadth. The Victoria Tubular Bridge is a corridor 16 ft. wide, and from 19 to 22 ft. high. The Britannia Bridge is 17 ft. wide, 19 ft. high at the ends, and 30 ft. high in the centre.

SPECIFIC GRAVITY.—The specific gravity of a body is its ratio of weight to that of an equal bulk of water. Water is heaviest at 39.83° Fahr., but in this table is taken as a standard at 60°.

Substances.	Specif. Gravity.	Substances.	Specif. Gravity.
Platinum,	21.	Sulphur,	2.03
Gold,	19.24	Bone,	2.
Tungsten,	17.	Ivory,	1.92
Mercury,	13.6	Caoutchouc,	.99
Rhodium,	11.	Sodium,	.97
Silver,	10.47	Wax,	.97
Bismuth,	9.82	Gutta-percha,	.966
Copper,	8.78	Ice,	.917
Arsenic,	8.60	Pumice-Stone,	.92
Steel,	7.81	Potassium,	.82
Wrought-Iron,	7.78	Pine Wood,	.66
Cast-Iron,	7.21	Poplar,	.38
Zinc,	6.86	Cork,	.34
Antimony,	6.71	LIQUIDS:	
Iodine,	4.95	Sulphuric Acid,	1.84
Heavy Spar,	4.43	Nitric Acid,	1.22
Ruby,	4.28	Milk,	1.03
Diamond,	3.50	Wine,	.99
Flint Glass,	3.33	Linseed Oil,	.94
Emerald,	2.77	Spirits Turpentine,	.87
Marble,	2.7	Alcohol, pure,	.79
Quartz,	2.65	Sulphuric Ether,	.72
Sulphate of Lime,	2.32	Naphtha, "light oil,"	.73

A cu. in. of distilled water weighs, at 62° Fahr. and 30 inches pressure, 252.456 grains.

As 10 avoir. lbs. constitutes the Eng. imperial gallon, that measure contains 277.274 cu. in.

The U. S. stand. gallon contains 58.372 grains of water at 39.83° Fahr. and 30 inches pressure.

BRICKWORK.—**Roughly estimated, 20 bricks=1 cu. foot.**

Front brick, Phila., . .	$8\frac{1}{4} \times 4\frac{1}{3} \times 2\frac{2}{3}$	Maine, . . .	$7\frac{1}{2} \times 3\frac{3}{8} \times 2\frac{3}{8}$
" " Croton, .	$8\frac{1}{2} \times 4 \times 2\frac{1}{4}$	Milwaukee, . .	$8\frac{1}{2} \times 4\frac{1}{8} \times 2\frac{3}{8}$
Fire brick, American,	$8\frac{7}{8} \times 4\frac{1}{2} \times 2\frac{5}{8}$	Common, . . .	$8 \times 4\frac{1}{8} \times 2\frac{1}{2}$

The mortar thickness is called the **joint.** The **volume** of a brick equals length and joint × width and joint × thickness and joint. The width and joint of a common brick in a 14-inch wall would be about 4.7 in. If the other joints were .25 in., then its volume, in such a wall, $=8\frac{1}{4} \times 4\frac{3}{8} \times 4\frac{7}{10}=54.285+32=17.14$. In fact, the joint is usually $\frac{1}{8}$ in. thick.

Masons, in estimating **work,** measure the outside of each wall, and make one-half allowance for doors and windows. Chimneys are measured solid, except the place below the mantel. Five courses are generally called a foot.

STONE MASONRY.—Quantities in cu. feet or in perches. A **perch** is $16\frac{1}{2}$ ft. long, 1 ft. wide, and $1\frac{1}{2}$ ft. thick ; in all, $24\frac{3}{4}$ cu. ft.—or, allowing $2\frac{3}{4}$ ft. for mortar and filling, a perch contains 22 ft. of stone. For **mortar,** there is allowed 3 pecks of lime and 4 bushels of sand. A wall 50 ft. by 11 ft. by 3 ft. $=\frac{50 \times 11 \times 3}{24.75}=66\frac{2}{3}$ perches.

MEASUREMENT OF LOGS.—**Common rule.**—Square one-third of the sum of the greatest and the least diameters in inches, multiply by the length of the log in feet, and divide by 144.

A log 12 ft. long, 24 in. in diameter at one end, and 15 in. at the other, would contain $14\frac{1}{12}$ cu. ft. $\left(\frac{24+15}{3}\right)^2 \times \frac{12}{144}=14\frac{1}{12}$. Seven-tenths of the diameter of a circle equals the side of the inscribed square. Hence the squared log will be about $\frac{2}{3}$ of the thickness of the smaller end.

The entire cubic contents of a round log equals the square of the circumference at the middle of the log, multiplied by eight times the length, divided by 100. Thus, a log 10 ft. long and 30 in. in circumference at the middle contains 720 cu. ft. ($30^2 \times 10 \times 8 \div 100=720$.)

By Doyle's Rule, the contents of a round log, *in square-edged boards,* may be found thus : From the diameter in inches of the smaller end within the bark subtract 4 ; square the remainder ; multiply the square by the length in feet ; and divide by 16.

SAWED LUMBER.—The unit for the measurement of sawed lumber is a **square foot one inch thick.**

Table Founded on Doyle's Rule.

The following indicates the number of feet, board-measure, in a log 10 feet long and of either of the diameters mentioned :

Diam. In.	Ft.	Diam. In.	Ft.	Diam. In.	Ft.	Diam. In.	Ft.
10	23	19	140	28	360	37	681
11	31	20	160	29	391	38	723
12	40	21	180	30	422	39	765
13	50	22	202	31	456	40	810
14	62	23	225	32	490	41	850
15	75	24	250	33	526	42	903
16	90	25	275	34	562	43	952
17	105	26	302	35	601	44	1000
18	122	27	330	36	640	45	1051

Examples: A log 14 ft. long, 13 in. in diam., contains 70 ft. board-measure, for by the table $5.0 \times 14' = 70$. A log 17 ft. long and 42 in. in diameter$=90.3$ ft. $\times 17 = 1535.1$ feet. The number opposite the diameter divided by 10 must be multiplied by the number indicating the actual length of the log in question.

GAUGING OF CASKS.

Rule.—*Take the interior length of the cask, the diameter at the bung, and the diameter at the head, all in inches. Subtract the head diameter from the bung diameter, and note the difference. If the staves are slightly curved, multiply this difference by .6 ; if much curved, multiply by .7. Add the product in either case to the head diameter to determine the mean diameter.*

The continued product of the square of the mean diameter, the length, and .0034 will equal the number of **wine gallons.** *The contents in* **litres** *will be expressed by the product of the square of the mean diameter, the length, and .0129.*

Note.—First reduce the cask, theoretically, to the form of a cylinder ; in other words, find its average, or *mean, diameter.* The area of a cylinder equals its height $\times$ diameter$^2 \times .7854$. If this is estimated in cubic inches, and the result is to be reduced to wine gallons, it must be divided by 231. But $.7854 \div 231 = .0034$. For litres divide by 61.027. But $.7854 \div 61.027 = .0129$.

ALLIGATION is the process of finding how much of several different ingredients is required to compose a mixture whose value is defined.

Problem.—A merchant has coffees worth respectively 9, 12, 24, and 30 cents a lb. What proportionate part of each must be taken to make up a package worth 20 cts. a lb.?

Rates.		Gains or losses.	3.	4.	5.	6.	7.	8.	9.	10.
20	9	+11	$\frac{1}{11}$	$\frac{40}{440}$	40	10	10	30	10	5
	12	+ 8	$\frac{1}{8}$	$\frac{55}{440}$	55	55	1	1	4	1
	24	— 4	$\frac{1}{4}$	$\frac{110}{440}$	110	110	2	6	2	1
	30	—10	$\frac{1}{10}$	$\frac{44}{440}$	44	11	11	11	44	11

Explanation.—The first and second columns need no explanation. The fractions in the third column show how much of each sort must be taken to gain or lose 1 cent. There being as many kinds cheaper than the mean rate as there are of those dearer, the gains balance the losses if the quantities taken are in proportion as the fractions. These reduced to their common denominators are in the ratio of the numbers in the fifth column.

Now if a multiple (or a sub-multiple) of one of the cheap kinds be taken, whether of drams, ounces, or pounds, a corresponding multiple (or sub-multiple) of a dear kind must be taken to offset it. With this restriction, an indefinite number of variations may be made from the combinations indicated by the simple fractions. (See columns 6, 7, 8, 9, and 10.)

It will be seen that the value of the 24 lbs., making the sum of column 7, are worth $4.80, or 20 cents per pound; etc.

LIFE INSURANCE.

The Insurer receives a contract, called a **policy**, which provides for the payment at his death of a certain sum to his heirs or assigns. For this he agrees to pay annually, or semi-annually, a certain sum, called a **premium.** *Net premiums* are calculated without reference to the expenses of a company. *Office premiums* are equal to the net premiums increased by a margin to cover office expenses. The premiums

paid year by year remain the same, but a person who buys a policy at 40 yrs. of age must pay more than one who takes out a policy at the age of 25. The company sets aside more and more of the premium annually paid in, as a party grows older, in order to meet the increased risk. Thus it is that the insured party seems to receive less and less in the way of dividends. **Dividends** are allowances returned to the policy-holder by any mutual company out of the profits of the company. The profits arise out of the margin first charged, out of the interest on the sums reserved, and out of the surplus vitality of policy-holders. (The death-rate is usually not so great as the actual estimate.)

The reserves set aside allow for the paying of what is called a **surrender value** upon application by a policy-holder.

The premiums of life insurance are generally reckoned as a certain sum on $100, payable annually in advance. The following is a brief table of the usual rates. Column A indicates the nearest birth-day at time of insurance:

A.	7 Years.	For Life.	A.	7 Years.	For Life.	A.	7 Years.	For Life.
24	1.07	1.98	34	1.50	2.64	44	1.94	3.63
25	1.12	2.04	35	1.53	2.75	45	1.96	3.73
26	1.17	2.11	36	1.57	2.81	46	1.98	3.87
27	1.23	2.17	37	1.63	2.90	47	1.99	4.01
28	1.28	2.24	38	1.70	3.05	48	2.02	4.17
29	1.35	2.31	39	1.76	3.11	49	2.04	4.49
30	1.36	2.36	40	1.83	3.20	50	2.09	4.60
31	1.42	2.43	41	1.88	3.31	51	2.20	4.75
32	1.46	2.50	42	1.89	3.40	52	2.37	4.90
33	1.48	2.57	43	1.92	3.51	53	2.59	5.24

The payment of $34 annually by a person who takes out a policy at the age of 42 will insure to his assigns $1000 at his death.

For more copious information, see encyclopædias and documents issued by Life Assurance Companies. Very elaborate treatises have been published on this subject.

A person whose age was 36 invested $158 annually, receiving compound interest. He lived 10 years. His heirs would receive $2335.80. If he had paid $157 annually in premiums he would have received $10,000, plus the usual additions. (See Table, page 178.)

What would have been the difference *against* him, if he had lived as long as the table on page 180 indicates?

A gentleman 68 years of age takes out a policy for $5000 at a premium of $500. He also invests $500 annually at 7% compound interest. If he live but 11 years, from which fund will his heirs receive the greatest sum.

NOTE.—$217 thus invested will yield $3664.80.

A comparison may be made by means of the following Table of the advantages afforded by savings deposited and of savings devoted to Life Insurance:

Compound Interest Table.

This table shows the amount that would result from an annual investment of one dollar at the end of each of any number of years, from one to thirty. Its relation to Life Insurance is obvious.

End of Year.	4 per cent.	5 per cent.	6 per cent.	7 per cent.
1	1.0400	1.0500	1.0600	1.0700
2	2.1216	2.1525	2.1836	2.2149
3	3.2465	3.3101	3.3746	3.4399
4	4.4163	4.5256	4.6371	4.7507
5	5.6330	5.8019	5.9753	6.1533
6	6.8983	7.1420	7.3938	7.6540
7	8.2142	8.5491	8.8975	9.2598
8	9.5828	10.0266	10.4913	10.9780
9	11.0061	11.5779	12.1808	12.8164
10	12.4864	13.2068	13.9716	14.7836
11	14.0258	14.9171	15.8699	16.8885
12	15.6268	16.7130	17.8821	19.1406
13	17.2919	18.5986	20.0151	21.5505
14	19.0236	20.5786	22.2760	24.1290
15	20.8245	22.6575	24.6725	26.8881
16	22.6975	24.8404	27.2129	29.8402
17	24.6454	27.1324	29.9057	32.9990
18	26.6712	29.5390	32.7600	36.3790
19	28.7781	32.0660	35.7856	39.9955
20	30.9692	34.7193	38.9927	43.8652
21	33.2480	37.5052	42.3923	48.0057
22	35.6179	40.4305	45.9958	52.4361
23	38.0826	43.5020	49.8156	57.1767
24	40.6459	46.7271	53.8645	62.2490
25	43.3117	50.1135	58.1564	67.6765
26	46.0842	53.6691	62.7058	73.4833
27	48.9676	57.4026	67.5281	79.6977
28	51.9663	61.3227	72.6398	86.3465
29	55.0849	65.4388	78.0582	93.4608
30	58.3283	69.7608	83.8017	101.0730

Example.—How much would $153, **annually** invested, at six per cent. compound interest, produce in fifteen years?

Rule.—Multiply 24.6725 (above) by 153.

Result.—$3,774.89.

Expectation of Life means the probable number of years remain-

ing to a person, calculated by a standard table. A well-known table is that prepared by a physician of Carlisle, Eng., based on statistics gathered between the years 1770–1790. That used in the U. S. by many companies was prepared by Dr. Wigglesworth, of Massachusetts.

Dr. Wigglesworth's Table.

Age.	Years.	Age.	Years.	Age.	Years.	Age.	Years.	Age.	Years.
1	36.78	16	35.76	31	29.83	46	23.37	61	14.86
2	38.74	17	35.37	32	29.43	47	22.83	62	14.26
3	40.01	18	34.98	33	29.02	48	22.27	63	13.66
4	40.73	19	34.59	34	28.62	49	21.72	64	13.05
5	40.88	20	34.22	35	28.22	50	21.17	65	12.43
6	40.69	21	33.84	36	27.78	51	20.61	66	11.96
7	40.47	22	33.46	37	27.34	52	20.05	67	11.48
8	40.14	23	33.08	38	26.91	53	19.49	68	11.01
9	39.72	24	32.70	39	26.47	54	18.92	69	10.50
10	39.23	25	32.33	40	26.04	55	18.35	70	10.06
11	38.64	26	31.93	41	25.61	56	17.78	71	9.60
12	38.02	27	31.50	42	25.19	57	17.20	72	9.14
13	37.41	28	31.08	43	24.77	58	16.63	73	8.69
14	36.79	29	30.66	44	24.35	59	16.04	74	8.25
15	36.17	30	30.25	45	23.92	60	15.45	75	7.83

A table such as the preceding is useful in computing annuities as well as in Life Assurance estimates.

1. What is the present worth of an annuity of $510 for 20 years, estimating compound interest at 5%? *Ans.* $6355.73.

2. What is the present value of an annuity of $468 for 16 years at 4½% compound interest? *Ans.* $5257.52.

3. How much ought to be given for the lease of a store to run for 27 years if the yearly rental is $740, estimating compound interest at 4%? *Ans.* $12084.52.

4. What must a person, whose "expectation of life" is 15 years, pay for a life annuity of $300, estimating money at 3%? *Ans.* $3581.38.

At 5%, what is the value of a lease which will expire on the death of a person now 45 years old, "the expectation of life" being 24 years?

What is the difference between the amount of an annuity of $150 for 8 years, compound interest at 6%, and the amount of the same if estimated at simple interest?

What is the value of $500 annuity forborne for 15 years at 6% compound interest? What of $270? Of $160?

What is the value of $300 annuity forborne 8 years at 5% compound interest? For 7 years at 7%?

What is the value of a lease for $1200 forborne for 4 years at 7% compound interest?

A person whose age is 54 years, and whose "expectation of life" is 19 years, wishes to purchase an annuity for $500. If money is worth 5%, what should he pay?

The present value of an annuity of $1 for 19 years at 5% is 12.085921. Of $500, it is $6042.96.

To Find the Present Value of a Deferred Annuity.

RULE.—*Find the difference between the present value of* $1 *from the present time till the* TERMINATION *of the annuity and the present value of* $1 *from the present time till the date of the* COMMENCEMENT *of the annuity. Multiply this difference by the given annuity.*

1–2. A man contemplates the purchase of a life estate in a property worth $1200 per annum. He will not take the property till after the lapse of 6 years, and will then live to enjoy it probably ten years. What should he pay, money being worth 6%? At 7%?

3–4. What is the present value of a leasehold of $1500 deferred 8 years, and to run 16 years, at 6% compound interest? At 4% compound interest?

To Find the Annuity which a Given Sum will Purchase.

RULE.—*Divide the given sum by the present value of an annuity for* $1, *for the given rate and time.* (See Table.)

1–2. If a man buy a lease running 20 years at 6% for $8000, what should be the annual rental? What if he pay $10000 for a lease to run 18 years, money being worth 10%?

3–5. What annuity will $4000 purchase for 9 years, money being worth 5%? What $7000 for 12 years, money being worth 4%? What $2900 for 6 years, at 7%?

ANNUITIES.—An **annuity** is a sum of money payable at yearly or other equal intervals. It may be the income arising from a leasehold, a pension, or from Government securities.

A **Certain Annuity** is one which is to last for a fixed term.

A **Contingent Annuity** is usually one which is to last during the lifetime of a certain person, its continuance being uncertain.

An **Immediate Annuity** becomes payable at once.

A **Reversionary** or **Deferred Annuity** is one beginning at a future period.

An **Annuity Foreborne** is one in which the payments are in arrears.

The amount of an annuity is the sum of all the payments, together with the interest on each payment from the time it became payable. Thus, if an annuity be regularly paid at the end of each year for 20 years, interest on the first payment accrues for 19 years; on the second, for 18 years; etc. On the last payment no interest is computed.

Suppose the interest is at 6%, and that the annuity is $1, then the last payment = $1; the payment of the year before is $1 + its interest for $1; of the year before this, $1 + the compound interest for two years; of the 12th year, $1 + the compound interest for eight years; etc.

The series thus formed may be represented thus :

20th yr.	19th yr.	18th yr.	17th yr.	. . .	2d yr.	1st yr.
\$1.	\$1.06.	$\$1.06^2$.	$\$1.06^3$.	. . .	$\$1.06^{18}$.	$\$1.06^{19}$.

The ratio of this geometrical progression is \$1.06.

The *sum* of the series is the *amount* of the annuity.

$$\frac{1.06^{20}-1}{1.06-1}=\frac{3.2071355-1}{1.06-1}=\$36.785591.$$

If the annuity had been \$1000, the amount would have been \$36,785.

THE AMOUNT OF \$1 ANNUITY FOR ANY GIVEN RATE AND TIME will equal the amount of \$1 at compound interest for the given time divided by the interest of \$1 for one year. It may be conveniently found thus :

RULE.—*Find from the table the amount of \$1 for the given time and rate; multiply this number by the annuity required.*

Table giving the Amount of \$1 Annuity.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	8%.	10%.
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0300	2.0350	2.0400	2.0500	2.0600	2.0700	2.0800	2.1000
3	3.0909	3.1062	3.1216	3.1525	3.1836	3.2149	3.2464	3.3100
4	4.1836	4.2149	4.2465	4.3101	4.3746	4.4399	4.5061	4.6410
5	5.3091	5.3625	5.4163	5.5256	5.6371	5.7507	5.8666	6.1051
6	6.4684	6.5502	6.6330	6.8019	6.9753	7.1533	7.3359	7.7156
7	7.6625	7.7794	7.8983	8.1420	8.3938	8.6540	8.9228	9.4872
8	8.8923	9.0517	9.2142	9.5491	9.8975	10.2598	10.6366	11.4359
9	10.1591	10.3685	10.5828	11.0266	11.4913	11.9780	12.4876	13.5795
10	11.4639	11.7314	12.0061	12.5779	13 1808	13.8164	14.4866	15.9374
11	12.8078	13.1420	13.4864	14.2068	14.9716	15.7836	16.6455	18.5312
12	14.1920	14.6020	15.0258	15.9171	16.8699	17.8885	18.9771	21.3843
13	15.6178	16.1130	16.6268	17.7130	18.8821	20.1406	21.4953	24.5227
14	17.0863	17.6770	18.2919	19.5986	21.0151	22.5505	24.2149	27.9750
15	18.5989	19.2957	20.0236	21.5786	23.2760	25.1290	27.1521	31.7725
16	20.1569	20.9710	21.8245	23.6575	25.6725	27.8881	30.3243	35.9497
17	21.7616	22.7050	23.6975	25.8404	28.2129	30.8402	33.7502	40.5447
18	23.4144	24.4997	25.6454	28.1324	30.9057	33.9990	37.4502	45.5992
19	25.1169	26.3572	27.6712	30.5390	33.7600	37.3790	41.4463	51.1591
20	26.8704	28.2797	29.7781	33.0660	36.7856	40:9955	45.7620	57.2750

The **Present Value** of an annuity of \$1 for any number of years less than 21 is shown by the table appended.

TO FIND WHAT SHOULD BE PAID FOR ANY GIVEN ANNUITY.

RULE.—*Multiply the present worth of an annuity of \$1 for the given time and rate by the number denoting the given annuity.*

The basis of this rule is the following : Divide \$1 by the sum to

which it would amount at compound interest for the given time; the quotient will equal the present value of $1. Subtract this result from unity, multiply this difference by the annuity, and divide the result by the interest on $1 for one year. PROBLEM.—Find the present value of an annuity of $500 for 18 years at 3½%. *By the common rule and table*, 13.1897 × $500=$6594.85. *By the theoretical rule*, $\frac{1}{1.035^{18}}$=$.53836114. $1−$.53836114=$.46163786. $.46163786 × 500 ÷ .035= $6594.826.

TABLE GIVING THE PRESENT WORTH OF $1 ANNUITY.

Yrs.	3%.	3½%.	4%.	5%.	6%.	7%.	8%.	10%.
1	0.9709	0.9662	0.9615	0.9524	0.9434	0.9346	0.9259	0.9091
2	1.9135	1.8997	1.8861	1.8594	1.8334	1.8080	1.7833	1.7355
3	2.8286	2.8016	2.7751	2.7232	2.6730	2.6243	2.5771	2.4869
4	3.7171	3.6731	3.6299	3.5460	3.4651	3.3872	3.3121	3.1699
5	4.5797	4.5151	4.4518	4.3295	4.2124	4.1002	3.9927	3.7908
6	5.4172	5.3286	5.2421	5.0757	4.9173	4.7665	4.6229	4.3553
7	6.2303	6.1145	6.0021	5.7864	5.5824	5.3893	5.2064	4.8684
8	7.0197	6.8740	6.7327	6.4632	6.2098	5.9713	5.7466	5.3349
9	7.7861	7.6077	7.4353	7.1078	6.8017	6.5152	6.2469	5.7590
10	8.5302	8.3166	8.1109	7.7217	7.3601	7.0236	6.7101	6.1446
11	9.2526	9.0016	8.7605	8.3064	7.8869	7.4987	7.1390	6.4951
12	9.9540	9.6633	9.3851	8.8633	8.3838	7.9427	7.5361	6.8137
13	10.6350	10.3027	9.9856	9.3936	8.8527	8.3577	7.9038	7.1034
14	11.2961	10.9205	10.5631	9.8986	9.2950	8.7455	8.2442	7.3667
15	11.9379	11.5174	11.1184	10.3797	9.7122	9.1079	8.5595	7.6061
16	12.5611	12.0941	11.6523	10.8378	10.1059	9.4466	8.8514	7.8237
17	13.1661	12.6513	12.1657	11.2741	10.4773	9.7632	9.1216	8.0216
18	13.7535	13.1897	12.6593	11.6896	10.8276	10.0591	9.3719	8.2014
19	14.3238	13.7098	13.1339	12.0853	11.1581	10.3356	9.6036	8.3649
20	14.8775	14.2124	13.5903	12.4622	11.4699	10.5940	9.8181	8.5136

This table will be found useful in computing the present worth, or "sum in hand," of a person's interest in an estate such as a lease, or a right of dower. Thus, a person 52 years of age having a half interest for life in a rental of $1000, offers to accept a sum in hand as full satisfaction of the same. The conditions of the problem arising from this case are as follows:

(1) At 52 years of age the expectation of life is 20 years. It is required to find the present value of an annuity of $500 for 20 years. The % may be 5%, 6%, or 7%.

(2) A widow 60 years of age has a right of dower in an estate worth $30,000. The present worth of her interest, if money be worth 6%, is the present worth of an annuity of $600 for 15.5 years, or about $5850. (See Tables p. 180 and p. 183.)

Table showing Rate of Interest allowed in each State, the Penalty for Usury, and the time at which a Claim ceases to be collectible.

State.	Legal rate.	Contr. rate.	Penalty.	Stat. of Lim. O. A.	Stat. of Lim. N.
Ala.,	8	8	*e. i.*	3	6
Ark.,	6	*any*		3	7
Cal.,	10	"		2	4
Conn.,	6	6	*e. i.*	6	17
Fla.,	8	*any*		5	5
Ga.,	7	10	*ex.*	3	3
Ill.,	6	10	*e. i.*	5	6
Ind.,	6	10	*ex.*	6	20
Iowa,	6	10	*e. i.*	5	10
Kan.,	7	12	*e. i.*	3	5
Ky.,	6	10	*ex.*	2	7
La.,	5	8	*e. i.*	3	5
Me.,	6	*any*		6	6
Md.,	6	6	*ex.*	3	3
Mass.,	6	*any*	{	20*w.* 6	20*w.* 6
Mich.,	7	10	*ex.*	6	6
Minn.,	7	12	*ex.*	6	6
Miss.,	6	10	*ex.*	3	6
Mo.,	6	10	*e. i.*	5	10
Neb.,	10	12	*e. i.*	4	5
Nev.,	10	*any*			

State.	Legal rate.	Contr. rate.	Penalty.	Stat. of Lim. O. A.	Stat. of Lim. N.
N. H.,	6	6	3 *ex.*	6	6
N. J.,	7	7	*e. i.*	6	16
N. Y.,	7	7	*e.i.v.*	6	6
N. C.,	6	8	*e. i.*	3	3
Ohio,	6	8	*ex.*	6	15
Or.,	10	12		6	6
Penn.,	6	*any*	{	6	20*s.* 6
R. I.,	6	*any*		6	6
S. C.,	7	*any*		6	6
Tenn.,	6	10	*ex.*	6	6
Texas,	8	*any*		2	4
Utah,	10	*any*			
Vt.,	6	6		6	14*w.* 6
Va.,	6	12	*ex.*	5	20*s.* 5
Wash.,	10	*any*			
W. Va.,	6	6	*ex.*	5	20*s.* 5
Wis.,	7	10	*e. i.*	10	6

ABBREVIATIONS.—*e. i.*, forfeiture of the entire interest ; *ex.*, excess ; *s.*, sealed ; O. A., open account ; *w.*, witnessed ; N., note ; *any*, any rate agreed on ; Contr. r., contract rate ; Stat. Lim., statute limitations ; *v.* (in N. Y.), usurious contracts are void. In Virginia, on proof of usury, the principal as well as the interest is forfeited. *Judgments* in the States of California, Colorado, Connecticut, Kansas, Maryland, Mississippi, Ohio, and Vermont are good for from 3 to 7 years ; in other States, for from 10 to 20 years. 20*w* indicates that a witnessed contract holds good for 20 years. Ditto 20*s*.

A statute of limitations is a State law by which the legal liability of a debtor ceases at the expiration of a certain time after the incurring of the debt. In general, however, a new promise before witnesses or in writing will revive the obligation for a full period from the time of the promise.

In computing interest, in order not to endanger his claim a creditor should never estimate days as 360ths of a year. By statute, in most States, days must be reckoned as 365ths of a year. Consider the time as calendar years, calendar months, and the extra days as 365ths. Thus, the interest on a note of $400 for 63 days is $4.14 and not, as is usually reckoned, $4.20.

Number of Days from any Day of any Month to the same Day of any Month not more than one Year later.

From	To Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
Jan., . .	365	31	59	90	120	151	181	212	243	273	304	334
Feb., . .	334	365	28	59	89	120	150	181	212	242	273	303
Mar., . .	306	337	365	31	61	92	122	153	184	214	245	275
Apr., . .	275	306	334	365	30	61	91	122	153	183	214	244
May, . .	245	276	304	335	365	31	61	92	123	153	184	214
June, . .	214	245	273	304	334	365	30	61	92	122	153	183
July, . .	184	215	243	274	304	335	365	31	62	92	123	153
Aug., . .	153	184	212	243	273	304	334	365	31	61	92	122
Sept., . .	122	153	181	212	242	273	303	334	365	30	61	91
Oct., . .	92	123	151	182	212	243	273	304	335	365	31	61
Nov., . .	61	92	120	151	181	212	242	273	304	334	365	30
Dec., . .	31	62	90	121	151	182	212	243	274	304	335	365

NOTE.—1 Add 1 day if the interval include Feb. of a leap year.

2. If the interval ends at a later or an earlier day of the month than that on which it begins, add or subtract accordingly. Thus, from May 7 to February 12=276+5 ; and June 4 to January 14=214+10.

STOCKS AND BONDS.

Capital Stock is the fund furnished by the individuals of a stock company.

Stockholders are the owners of shares of stock.

Stocks are the certificates of the proper officers of a stock company issued to stockholders showing the number of shares of stock owned by each.

Stocks common and preferred.—Owners of common stocks can claim only such dividends as are left after the claims of owners of preferred stocks are paid.

Scrip is a certificate of dividend when not paid in cash, and is generally convertible into capital stock. Bonds are written obligations secured by mortgages on real property generally issued by stock companies pledging the payment of sums of $100, $1000, or $5000 after a specified time.

Government Bonds are certificates of the indebtedness of Government. They are, in fact, stocks, as they are not secured by any actual

mortgage. The interest is paid semi-annually in coin. The words 5-20 or 10–40 signify that the U. S. may at its option redeem the bonds in from 5 to 20 years, or from 10 to 40 years, from their date. Sixes of 1881 (quoted "6s of '81") bear 6% interest, and are payable in 1881. U. S. 6s 5-20 of '85, means bonds that may be paid in from 5 to 20 yrs., and which are due in 1885. **Coupons** are interest checks attached to bonds. These are payable to bearer, and, after they fall due, draw interest like a note until they are paid. Registered bonds have no coupons. On these the interest is payable only to the owner as registered at the Treasury Department, Washington, D. C.

Consols are U. S. bonds issued in 1865–'67 and '68, when a portion of the National Debt was refunded and consolidated. *Consols* was at one time confined in its meaning to *English 3% Government bonds.*

Stock quotations are the statements of current values as published by such great associations as the London Stock Exchange, the New York Stock Exchange, and the Paris Bourse.

STOCKBROKERS' TERMS.—Buyer's option, as "buyer 5, 10, or 20 days," is the buyer's privilege to demand the delivery of stocks purchased within the time specified. It also obliges him to take them on the last of the specified number of days. **Seller's option** confers the privilege of delivering within the specified time, and imposes the obligation of delivery on the last day. **Puts** are like "sellers' options," except that there is no obligation to deliver on the last of the specified days. **Calls** are like buyers' options, divested of the obligation to take the stock on the last day. A broker who buys a "put" is a "**bear**"; if he purchase a call he is a "**bull.**"

A **Bull** is a dealer whose profits must arise from an advance in prices. A **Bear** is a dealer who wishes to depress the price of stocks, etc. He is a dealer who "**sells short**"—*i.e.*, sells what he has not yet bought. He expects that, before he is required to deliver it, he will be able to buy it at a lower rate than he has already sold it at.

A **Corner** is made when dealers secure control of all, or nearly all, the stock of a company, and then advance its price, so that those who have "sold short" must, in order to fulfil their contracts, purchase at a loss.

The agents or brokers who buy or sell for others are usually members of the Government Stock Board, or of the New York Stock Exchange. Persons who employ them, and who buy on speculation, are required by the brokers to deposit from 5% to 10% on the par value of the stock bought. This deposit is called a **margin.** **Investment stocks** are such as usually pay dividends, such as the bonds of cities, States, or of the nation.

The **brokerage** charged for buying or selling stocks is from $\frac{1}{8}$% to $\frac{1}{4}$% of the par value.

CUSTOM-HOUSE BUSINESS.

DUTIES are **Government assessments on imported goods.** They are collected at offices called Custom-Houses.

Specific Duties are assessments by the quantity: a certain sum per box, bale, or lb.

Ad valorem duties are percentages determined by the market value of goods at the place of purchase, as shown by the invoice.

An Invoice or Manifest is a bill of particulars descriptive of goods, and stating their cost in the money of the country where purchased, certified by the nearest U. S. consul, other foreign consul, or by two respectable merchants. One invoice must not embrace goods shipped in different vessels. At London, Manchester, Leeds, Glasgow, Belfast, Paris, Lyons, Zurich, Basle, Aix-la-Chapelle, Berlin, Leipsic, Dresden, Vienna, Frankfort, and Brussels samples of the goods may be required for deposit by the consuls. The oath of a shipper may also be required. A copy of this invoice, properly sealed, is then forwarded to the collector of the port of entry, who will cause it to be compared with that presented by the importer when the goods are relieved from bond—*i.e.*, when the duties are paid. Another copy is given to the shipper, and a third is filed in the consular office.

Tare is an allowance made for the bag, box, or barrel in which goods are contained.

Actual Tare is allowed by law. This is generally 2% for goods in sacks, from 4% to 8% for light goods in mats and double bags, 10% to 12% for heavy goods in casks, and from 14% to 25% for light goods, such as raisins, in boxes.

Draft, or a deduction for waste, is not allowable. Tare is not allowed in adjusting *ad valorem* duties.

The Collector of the Port supervises all entries and receives all moneys. He employs all weighers, gaugers, and measurers.

Most merchants employ a **Custom-House broker** as an agent to attend to the complex details incident to the entry of goods—that is, to the passing of the papers relating to them through the Custom-House. The broker's commission is usually from 2% to 4% on the duties and charges paid.

A Tariff is the schedule of the legal rates of duties. When the rates are so high as greatly to check foreign competition, the tariff will operate more to *protect* domestic manufactures than to afford a large *revenue.*

A Clearance is a certificate given by the collector of a port that the vessel has been entered or cleared according to law—that is, that the master has lodged the proper papers in the Custom-House.

Value of Foreign Coins in U. S. Money (gold), January 1, 1877.

Country.	Denomination.	U. S.	Country.	Denomination.	U. S.
Arg. Rep.,	Peso fuerte,	$1.00	Japan,	Yen,	$.99,7
Austria,	Florin,	.45,3	Liberia,	Dollar,	1.00
Belgium,	Franc,	.19,3	Mexico,	Dollar,	.99,8
Bolivia,	Dollar,	.96,5	Netherl'ds,	Florin,	.38,5
Brazil,	Milreis,[1]	.54,5	Norway,	Crown,	.26,8
Bogota,	Peso,	.91,2	Paraguay,	Peso,	1.00
Canada,	Dollar,	1.00	Peru,	Dollar,	.91,8
Cent. Am.,	Dollar,	.91,8	Porto Rico,	Peso,	.92,5
Chili,	Peso,	.91,2	Portugal,	Milreis,[4]	1.08,4
Cuba,	Peso,	.92,5	Russia,	Rouble,[5]	.73,4
Denmark,	Crown,	.26,8	Sand. Isl.,	Dollar,	1.00
Ecuador,	Dollar,	.91,8	Spain,	Peseta,[6]	.19,3
Egypt,	Pound,[2]	4.97,4	Sweden,	Crown,	.26,8
France,	Franc,	.19,3	Switz'rl'd,	Franc,	.19,3
Great Brit.,	Pound sterl.,	4.86.6½	Tripoli,	Mahbub,[7]	.82,9
Greece.	Drachma,	.19,3	Tunis,	Piaster,[8]	.11,8
Germ. Em.,	Mark,	.23,8	Turkey,	Piaster,	.04,3
Hayti,	Dollar,	.95,2	U. S. Col.,	Peso,	.91,8
India,	Rupee,[3]	.43,6	Uruguay,	Patacon,	.94,9
Italy,	Lira,	.19,3			

[1] Equals 1,000 reis ; [2] 100 piasters ; [3] 16 annas ; [4] 1,000 reis ; [5] 100 copecks ; [6] 100 centimes ; [7] 20 piasters ; [8] 16 caroubs.

The Mint charge for recoinage is one-fifth of one per cent. Silver coins are estimated at $1.20 per oz., 900 fine. The Japan trade dollar, silver, weighs 420 grs., 900 fine. In the U. S., silver U. S. coins are legal tender for payments of $5 or less.

FREIGHT AND STORAGE.

Freight is the cost of transporting goods. Sometimes it implies the goods transported.

Storage is the keeping of goods for another. It generally signifies the sum charged for the care and protection of goods. It is estimated

for short periods, as for one week or one month. Storage is charged for an entire period, though goods are kept for only a fraction of a first period, but in some cities *for less than half* of a subsequent period no charge is made. The storage is paid when the goods are taken out of the warehouse. If several receipts have been taken for things stored at different times, and only a part of the goods are required, it should be taken out on the oldest receipt, or as much of it as possible. Thus: If a warehouseman received, Jan. 7, 600 casks of wine, and, Jan. 28, 500 casks, and delivered on Jan. 24 450 casks, and Feb. 4 delivered 250 casks, and Feb. 8 delivered 400 casks, his charges being 6c. per cask for the first 10 days or fraction, and 4c. for each additional 10 days or fraction, his account may appear as follows:

Date.	Received.	Delivered.	Paid.
Jan. 7, . .	600		
" 24, . .		450	$63
" 28, . .	500		
Feb. 4, . .		250	$37
" 7, . .		400	$40

EXPLANATION.—On the first lot withdrawn, viz., Jan. 24, 450 casks, the charges were for 10 days + 7 days, or (10c. + 4c.) × 450 = $63. On 150 casks out of the 250 withdrawn Feb. 4, the charges were (10c. + 4c. + 4c.) × 150, or $27, and on the other 100 casks withdrawn at the same time the charges were 10c. × 100, or $10, making together $37. The last item refers to the storage of 400 casks from Jan. 28 to Feb. 7, these 400 being a part of the 500 receipted for at that time.

NOTE.—A full exposition of this subject would exceed the space at our disposal. The customs of different localities sometimes make the necessary calculations very intricate.

RAILROAD FREIGHTS.—A car-load is estimated at 18000 lbs. to 20000 lbs.

A barrel of each of the following is estimated to weigh as follows:

Ale or beer, .	320 lbs.	Salt, fine, . .	300 lbs.	Hemp-seed, bu.,	44 lbs.
Apples, . .	150 "	" coarse, .	350 "	Malt, . . "	38 "
Beef, . . .	320 "	Vinegar, . .	350 "	Oats, . . "	32 "
Cider, . .	350 "	Whiskey, . .	350 "	Onions, . "	57 "
Corn-meal, .	220 "	Apples, . bu.,	56 "	Potatoes, . "	60 "
Eggs, . . .	200 "	" dried, "	24 "	" sweet, "	55 "
Fish, . . .	300 "	Barley, . "	48 "	Rye, . . "	56 "
Flour, . . .	200 "	Beans, . "	60 "	Salt, fine, . "	56 "
Spirits, . .	350 "	Buckwheat, "	52 "	Turnips, . "	56 "
Lime, . . .	200 "	Clover-seed, "	60 "	Wheat, . "	60 "
Pork, . . .	320 "	Corn in ear, "	70 "	Nails, . keg,	108 "
Potatoes, . .	150 "	Flax-seed, "	56 "	Salt, . sack,	200 "

A car-load of pine, poplar, or hemlock is	. . . . .	6.5 M.
" " black-walnut, ash, cherry, or maple,	. .	5 M.
" " unseasoned pine, poplar, or hemlock,	. .	5 M.
" " " black-wal., ash, cherry, or maple,	.	4 M.
" " seasoned oak, hickory, or elm,	. . . . .	5 M.
" " unseasoned oak, hickory, or elm,	. . . .	4 M.
" " " shingles,	. . . .	55 M.
" " lath,	. . . .	40 M.
" " brick,	. . . .	5 M.
" " stone, cubic yards,	. . . .	5
" " coal, bushels,	. . . .	250

Remarks on the Tables.

In 1760 Mr. Bird of London prepared a **yard measure** from a pendulum graduated at 39.1393 inches. Copies of this, or copies of copies, are in use throughout the British Empire and the United States. A commission of the members of the French Academy of Sciences developed the decimal system of weights and measures known as the metric system. They proposed to take as a unit the ten-millionth of the distance from the Equator to the Pole. The result of their measurements was **the metre,** a measure equal to 39.37079 English inches, or 39.368505 American inches United States Coast Survey.

Railroad engineers record their measurements in miles and feet. They use a 100-foot chain of one-foot links. An acre is not a tabular square, but is neatly expressed as 10 square chains. In 1866, the United States Government legalized the entire metric system. A **square metre** equals 10.7643 sq. ft. An are, 1076.43 sq. ft. The old Roman foot equalled 11.62 inches. At one time in Europe there were 130 different kinds of foot-measures.

Measures of capacity are obviously derived from linear measures, as are measures of weight. The cubic foot is equal to 2200 cylindrical inches, 3300 spherical inches, or 6600 conical inches. A cu. in. of pure water weighs 252.458 grains. A cu. ft. weighs 62.3457 avoir. lbs. The U. S. gallon of water weighs 58333 grs.; the British gal., 70000 grains. The U. S. bu. should contain 77⅞ lbs. of water.

The unit of weight is derived from a **standard Troy pound** prepared A.D. 1266. The American copies are derived from a copy of the English Imperial Troy Pound, made by Capt. Kater for the U. S. Mint at Phila., in 1827. **The avoir du pois pound** is to the Troy pound as 175 to 144, and contains 7000 grains. The French unit of weight, **the gramme,** represents a cubic centimetre of water in a vacuum at 39°.2 Fahr. It weighs 15.43242 English grains.

In very early times (Henry III. of England), the **merchants' pound** equalled 7680 grains, reckoning 32 grains to the dwt. **The carat** is one-tenth of this old dwt. Apothecaries' weight is also a relic of this old pound, an oz. apoth. being $\frac{1}{16}$th of 7680 grains.

COINS.—One cent weighs 48 grains. The **nickel 5-cent piece** weighs 5 grammes ; the **half-dollar,** 192 grains ; the **trade dollar,** 420 grains ; the **gold dollar,** 25.8 grains ; and other gold coins in proportion. The **Remedy of the mint**, or allowance for deviation from the exact standard of weight and fineness, is 12 grains to the lb. of gold in weight, and $\frac{1}{16}$ of a carat in fineness ; and of silver, 1 dwt. to the lb. in weight, and the same in fineness.

The **alloy** of the copper cent is 5% of tin or of zinc ; of the silver coins, 10% of copper ; of the gold coins, 1% of silver and 9% of copper. An oz. Troy of pure gold is worth $20.672 ; of pure silver, $1.293 ; of standard silver, $1.164. A sovereign contains $113\frac{1}{623}$ grains of pure gold ; the alloy is $\frac{1}{[illegible]}$.

MISCELLANEOUS PROBLEMS.

1. What is the value of a pile of wood 13 ft. long, 4.2 ft. wide, and 6.1 ft. high, at $3.625 a cord ?

2. Find the least number which is divisible by 9 without a remainder, and which when divided by 7 leaves 4 as a remainder.

3. A farmer desired to ascertain the distance from his house to the market-place in town. He marked one of his wagon wheels, and observed that it made 1924 revolutions during the journey thither. The wheel was 5 feet in diameter. What was the distance ?

4. Divide 276 into 4 such parts that the first shall be to the second as 2 to 5, the second to the third as 3 to 4, and the third to the fourth as 5 to 7. **(Distributive Proportion.)**

Explanation: The common multiple of 5 and 3 is 15. The ratio 2 : 5 =the ratio 6 : 15 ; the ratio 3 : 4=the ratio 15 : 20 ; the ratio 5 : 7=the ratio 20 : 28. Hence the four numbers are related as 6, 15, 20, 28. These equal 69. Let 276 be divided into 69 parts, then 6 of these parts, or 24, will equal the first number ; etc.

5. The pulse of a certain man beats 75 times in a minute, and 2 ounces of blood are expelled from the heart at each contraction. The man weighs 175 pounds, and his blood constitutes one-fifth of his entire weight. In what time will his blood circulate once through his heart ? How many tons of blood will his heart lift every 24 hours ?

6. If a cow yield 12 quarts of milk a day for 240 days, and 15 quarts of milk make 1 lb. of butter, how many lbs. of butter will be made in the season ?

7. The trees in a certain orchard stand a rod apart, and those next the boundary stand a half rod within. The orchard contains 10 acres ; the planting of each tree cost 63 cents ; what was expended in planting the orchard ?

8. A certain well is 30 feet deep ; its stone walls are 1.5 feet thick ; its interior diameter is 3.5 feet ; how many cubic yards of stone do the walls contain ?

9. A fish was immersed in some water contained in a pail 12 inches in diameter; the water rose .75 of an inch. Required the *weight* of the fish.

10. Some cloth was bought at $4.16 a yard on 8 mo. credit, and was sold immediately at $3.90 cash. Reckoning interest at 6%, what was gained or lost %?

11. A man, whose son and daughter were residing in Europe, provided in his will that if only the daughter returned his widow should have $\frac{3}{5}$ of his estate, but if only the son returned she should have $\frac{1}{4}$. Both son and daughter returned, consequently the widow received $2400 less than her portion would have been if the daughter only had returned. What would have been his mother's portion if only the son had returned? *Ans.* $2100.

12. Required the length of the sides of a square acre, also the diagonal of a plot 16 rods by 10 rods. How far does a person walk in ploughing an acre if the furrows are 1 foot wide?

13. There is a circular field 30 rods in diameter; what is the difference between its area and that of the circumscribed square; and between the areas of inscribed and circumscribed squares?

14. A debtor in London, wishing to pay a creditor in St. Petersburg 920 roubles, remits through Paris. He pays his broker at a time when exchange on Paris is 25.15 francs for £1, and that on St. Petersburg is 1 rouble for 1.20 francs. The broker delays the remittance until the rates are 25.35 francs for £1, and 1 rouble for 1.15 francs. What does the broker gain or lose by this delay?

15. Required the weight of the air in a room 8 yards long, 6 wide, and 4 high, the specific gravity of air being .0013?

16. Required the weight of an oak plank 10 feet long, 2 feet wide, and 2.5 inches thick, the specific gravity of oak being .9264?

17. Required the weight of a plank 12 feet long, 1.5 feet wide, and 2.25 inches thick, the material being ash? What if the wood were cedar? What if it were white pine? Chestnut? Hickory? (See Article 399.)

18. When cotton is quoted in New Orleans at 15 cents a pound, what should it be worth in Liverpool, exchange being quoted at 105, and freight 1.2 cents a pound?

19. When exchange was 484, a Bostonian sold U. S. 5s at 102, and, realizing $16,000 gold, invested the amount in British 3% consols at 91; what was his annual loss of income if the interest in English currency was remitted to him at par? *Ans.* $784.31 less $530.36, or $253.95.

20. If a man sells five $5000 bonds (U. S. 5-20s, gold bearing) at 116, and invests the proceeds in bonds and mortgages at 7% gold, how much will he add to his annual income?

21. Transfer $10,000 of 5s at 104 to 4s at 98, and find the difference in the annual income.

22. A merchant purchased goods as follows: Jan. 15, $500; Feb.

15, \$1000; March 9, \$1500. He received 6 months' credit on each item. On July 1, he gave his note for \$3000. At what time must it be paid?

23. A man sold a farm for \$5795, to be paid $\frac{1}{4}$ in 8 months and the remainder in three equal annual instalments. The parties finally agreed that the whole should be paid at once. What was the equated time?

24. If one man buys a bill of goods amounting to \$840 on a credit of 9 months, and another person purchases goods of equal value for cash, what should he pay, money being worth 7%?

25. A New-Yorker, travelling by way of London, Paris, Madrid, Lisbon, Genoa, Trieste, Cairo, Bengal, Canton, and San Francisco, bought bills on each city. He started with \$10,000. If he did not expend any of the proceeds, what sums in coin did he possess as he moved on, the £ being valued at \$4.87, the franc at \$.193, the doblon at \$5.02, the milrea at \$1.12, the lira at \$.193, the sequin (Egyptian) at \$2.50, the rupee at \$.445, and the tale (Chinese) at \$1.48?

26. In the composition of a quantity of gunpowder, the nitre was 10 lbs. more than $\frac{2}{3}$ of the whole; the sulphur, $4\frac{1}{3}$ lbs. less than $\frac{1}{6}$ of the whole; the charcoal, 2 lbs. less than $\frac{1}{7}$ of the nitre. What was the weight of the gunpowder?

27. A., B., and C. start together and travel the same way around an island 150 miles in circumference. A. goes 25 miles a day, B. 30 miles, and C. 50 miles. In what time will they all come together again at the point from which they started? How many miles will each have travelled?

28. What is the smallest sum which a person could have spent if he bought peaches at 2 cents, apples at 3 cents, pears at 4 cents, apricots at 5 cents, lemons at 6 cents, and oranges at 9 cents, and paid the same amount of money for each kind of fruit. How many of each kind did he buy?

29. In a certain college the Seniors number 144 students; the Juniors, 96; the Sophomores, 120; and the Freshmen, 216. It is required to arrange them in the largest companies possible without mingling different classes. What must be the size of each company?

30. One of two fields is 35.5 rods long and 27.4 rods wide; the other is 75.5 rods long and 68.8 rods wide. The first is valued at \$48; the second at 620 francs. Which is the cheaper?

31. A certain field is 145 metres long and 122 metres wide; the field adjoining is 200 metres long and 128 metres wide. If the field first described is worth 500 francs, how many dollars is the other field worth?

32. Required the height of a cubical box which will contain a globe of iron that weighs 100 lbs.

NOTE.—A cu. in. of cast-iron weighs .2607 lbs. A sphere equals in volume .5236 of a cube of equal height.

$$\textit{Solution:}\ \sqrt[3]{\frac{100 \div .2607}{.52367}} = \text{height of box.}$$

33. A house is to be built of the following dimensions: Length 40 ft., breadth 30 ft., height of the eaves 20 ft., perpendicular height of the gable ends 16 ft., walls 12 inches thick. Allowing 30 places for windows, each 6 ft. by 4 ft., and 5 places for doors, each 7 ft. by 4 ft., how many bricks will be required? (A brick with mortar occupies about 80 cu. in.) *Ans.* 49788.

34. A garden 120 yards by 100 yards is surrounded by a wall 2 ft. thick and 6 ft. 2½ in. high. The body of the wall is of stone; the facing is of brick 4 in. thick; the brick top-facing is 2½ in. thick. How many bricks, and how many cubic feet of stone, does the wall contain?

Ans. 10496 cu. ft. of stone, 125164 bricks.

35. How many shingles laid 5½ inches to the weather will cover a building 48 feet long, each side of the roof being 21 feet in width?

36. How many solid feet in a thorn-bush which being cut up and immersed in a vat 9 ft. long and 2¼ ft. wide caused the water to rise 8 in.?

37. A note drawn May 4, 1871, for $400 with interest at 7%, was made payable Jan. 10, 1874. A note-broker purchased this bill Feb. 1, 1872. What did he pay for it if the investment yielded him 8% interest? What would he have paid for it, if the words "with interest" had not been written in the note? What would he have paid for it under this last-mentioned condition, if he had purchased it Jan. 10, 1873?

38. A surveyor, in running a line, sighted a tree on the opposite side of a deep lake. Setting a stake on the line at the shore and another at 20 rods back, he measured off 10 rods at right angles from this last, and, again sighting the tree, had another stake set at the shore on this new line. The shore stakes were 110 feet apart. What was the breadth of the lake?

39. If the earth's diameter be estimated at 7900 miles, by what fraction of an inch would a mountain 4.5 miles high be represented on a globe 18 inches in diameter?

40. A person travelling "post" rode 92 miles in Prussia, 8100 yards to the mile; 3000 versts in Russia, 1166.7 yards to the verst; and 500 parasangs in Persia, 6076 yards to the parasang. How many miles did he traverse during this journey?

41. Find the sum of the series $4+11+18$ to 9 terms.

42. Find the sum of the series $3+6+12$ to 16 terms.

43. How long a wire .1 inch in diameter will 10 lbs. of copper make?

44. At what price must leather which cost 52 cents a lb. be sold in order to gain 25%?

VOCABULARY.

Acceptance, a formal agreement to pay a bill or draft.
Account, a systematic statement of debits and credits.
Acknowledgment, a formal avowal of one's signature.
Administrator, one who manages the estate of an intestate.
Affidavit, a formal statement sworn to before a qualified official.
Agio, the difference between the real and nominal value of money.
Aliquants, the parts of a number, *e.g.*, 14+7+3=24.
Aliquots, equal parts ; the quotients arising from integral divisors.
Analysis, an orderly and complete exposition of facts and principles.
Angle, the opening between two lines which meet.
Appraisal, the valuation set upon property of any kind.
Area, the surface included within any given lines.
Arithmetic treats of the units, relations, and properties of numbers.
Arithmetical mean of several numbers is the *average* (see Average).
Assessment, a valuation of property for purposes of taxation.
Assets, resources, or available or salable property.
Average equals the sum of addends given divided by their number.
Average, the time for the one payment of items due at different dates.
Avoirdupois, the system of weights for coarse articles.

Balance, the difference between the sides of an account.
Bankrupt, one who has been declared by a court to be unable to pay his debts.
Bill, a general name for a note, draft, or check.
Bill, a written account of things sold given to the buyer.
Bond, a sealed engagement to perform a contract.
Brokerage, the compensation paid for buying or selling stocks, etc.

Cambistry, the science of exchange, weights, and measures.
Capital, effects, such as cash or goods invested in a business.
Characters, #, @, =, %, %, meaning No., at, equals, per cent, account.
Check, an order drawn on a banker for the payment of money.
Clearance, the permit entitling a ship to leave a port.
Coin, metallic money, usually gold, silver, nickel, or copper.
Commercial discount, a deduction from the marked price of goods.
Commercial value, purchasing power or market price.
Common, belonging equally to more than one ; as, common divisor.
Complement, the difference between a number and its next higher unit.
Composite number, the product of two or more integers.
Consignee, a person to whom goods are sent for sale or care.
Consignment, goods sent to a consignee.
Contents, bulk, capacity, solidity, or volume.

Contract, an agreement between (two or more) parties competent to contract based on a sufficient consideration to do (or not to do) some legal act.
Conveyance, a writing for the legal transfer of property, as land.
Copartnership, a joint interest in business matters.
Coupons, interest warrants attached to public stocks.
Currency, coin and bills in circulation as money.
Customs, the legal tariff on imports.

Days of grace, legal overtime allowed the payer of a bill.
Diagonal, a line joining the vertices of two angles not adjacent.
Distributive proportion, partitive proportion.
Dividend, profits divided among stockholders.
Drachm, one-eighth of a fluid ounce.
Draft, a money order, an allowance for waste formerly made.

Exchange, premium or discount in the purchase of drafts.

Face, the amount expressed in a note or draft.
Factor, an agent to whom goods are entrusted for sale.
Factor, one of the exact integral divisors of a number.
Favor, advantage ; a draft is *in favor* of the payee.
Finance, the public revenue ; money matters in general.
Folio, the two pages seen when a book is opened.
Foreclosure, the cutting off of a mortgagor's right of redemption.
Frustum, that part of a pyramid left after removing the point.

Gallon, standard=8.339 avoir. of water=cyl. 7 in. in diam. 8 in. deep.
Gauging, measurement of casks, barrels, etc.
Guaranty, a warrantee by a person that another shall fulfil a contract.

Hypothecate, to pledge something as a collateral security.

Illegal, not sanctioned by law, as usury.
Imports, goods brought from a foreign country.
Insolvent, one whose assets are less than his liabilities.
Instalment, a partial payment.
Insurance, the premium payment for security against loss.
Integer, a whole number, a number not fractional.
Interest, the percentage charged for the use of money.
Intestate, one who dies leaving no will.
Intrinsic value, substantial worth, irrespective of form.
Investment, the money expended in the purchase of property.
Invoice, a bill of goods bought or sold sent to the purchaser.

Jobber, one who buys of importers and sells to retailers.
Joint-Stock, property held in common by a company.

Lateral, pertaining to the side; lateral strength, across the grain.
Law, a command or prohibition made by government.
Lease, a contract of rental of real estate.
Lien, a legal claim on property for something done to it.
Logarithm, that power of 10 which equals the given number.
Log-line, one knot=6.5 fathoms; knots per half min. show mi. per hr.

Magnitude, extent; that which can be measured.
Mathematics treats of the relations of magnitudes; it includes Arith., Geom., and Alg.
Maturity, the date at which a note or draft is payable
Minim, a drop of pure water, .95 of a grain Troy.
Money, tokens of value, as bills or coins.
Mortgage, to pledge property as security for a debt.
Multiple, a number which exactly contains a given number more than once.

Negotiable, transferable, with or without indorsement.
Negotiable words, pay to A. B., or *order* or *bearer*
Net, clear of all charges and reductions.
Notary Public, one who attests commercial writings to authenticate them abroad.
Note, a written promise to pay a specified sum of money.

Obligation, a bond with a condition and a penalty.

Parallelogram, a four-sided figure whose opposite sides are parallel.
Parallelopiped, a prism whose bases are parallelograms.
Partitive proportion relates to numbers proportioned as given numbers. (See Example 4, page 191.)
Par value, the value of stock as stated in the certificate.
Permutation, the different ways in which a number of things may be placed. The permutations of the digits $=1\times2\times3\times4$, etc. $=362880$.
Policy of Insurance, the contract between the insurer and the insured.
Practice, computations by the use of aliquots, aliquants, or ratios.
Premium, the sum paid for indemnity against loss.
Premium, the excess above par of stocks or drafts.
Principal, the sum loaned at interest: the employer of an agent.
Protest, a formal notice to sureties or endorsers.

Quotations, market value of stocks or general commodities.

Range, a row of townships 6 mi. wide, extending N. and S.—*Webster.*
Rebate, reduction for prompt payment.
Reciprocal, the quotient obtained by dividing 1 by a given number.
Rectangle, a four-sided figure having four right angles.
Retailer, one who sells to consumers.

Section, six hundred and forty acres of land.
Sight, at, as soon as presented (at 60 days, after sixty days).
Solve, to satisfy the conditions of a problem.
Solvent, able to pay all one's debts.
Specific Gravity, weight of a substance compared with water.
Stocks, interest-bearing gov't obligations ; shares of corporations.

Table, a collection of numbers referring to a common standard ; a list.
Tare, discount for the weight of bags, boxes, etc., containing mdse.
Tariff, a list of prices ; duties on imports.
Ton, 2000 lbs. ; Custom-house, 2240 lbs. ; French commercial, 2158.4.
Tonnage, carrying capacity of a ship ; duty on a ship.

Unit, a thing considered as a whole.
Unity, oneness.
Usury, excess above legal interest. (See Table.)

Vend, to sell ; **Vendee**, a buyer ; **Vender**, a seller.
Vertex, the point in which the sides of an angle meet.

Wharfage, charge for the use of a wharf.

INDEX.

PAGE

ABBREVIATIONS and Signs, 24, 51, 167
Accounts, averaging, . . . 105
" current, . . . 102
" kept in dollars and cents, . . . 50
Addends, ... 13
Addition of Integers, . . 13
" of Fractions, . . 38
" of Decimals, . . 48
" by tens, . . . 14
Ad valorem duties, . . 187
Algebraic forms, . . . 167
Aliquot parts of a year, . 67
Alligation, ... 176
Analysis, ... 117
Annuities, ... 181
Appendix, ... 167
Arc, ... 136, 143
Areas, Mensuration, . . 144
Arithmetic and Algebra, . 167
Arithmetical Progression, . 159
Assessments, ... 98
Averaging accounts, . . 102
Axioms, I.-VIII., . . . 13, 21

BALANCE of an account, . 102
" Interest, . . 102
" Cash, . . . 102
Bank, ... 74
" Discount, . . . 75
Base, ... 61, 65
Beams and Girders, . . 174
Bills of goods, ... 23
" of Exchange, . . . 85

PAGE

Bonds and Stocks, . . . 91
Brokers and Brokerage, 95, 186
Bulls and Bears, . . . 186

CALLS, Stockbroker's, . . 186
Cancellation, ... 22
Capital stock, ... 185
Cask, ... 154
Casting out Nines, . . . 27
Cipher, ... 10
Circle, ... 136, 143-147
Clock Time, ... 113
Coins, ... 87, 191
Commercial Discount, . . 79
Commission, ... 95
Common Denominator, . . 37
" Difference, . . 160
" Divisor, . . . 29
" Fractions, . . 34
" Multiple, . . . 30
Comparison of Fractions, . 40
" of Weights, 111, 187, 190
" of Volumes, . 133, 158
Complement Number, . . 131
Complex Fraction, . . 40, 46
Composite Number, . 18, 22, 25
Compound Interest, . 81, 179
" Number, . . 51
Cones, ... 150, 155, 156
Contract, ... 196
Corner, Stockbroker's . . 186
Correspondence of Division and Multiplication, . . 20
Cube, ... 152
Cube Root, ... 128

	PAGE
Currency, Eng., Fr., Ger.,	51, 54
Custom-House,	187
Cylinder,	153
DAYS between Dates,	185
Decimals,	42
" from Com. Frac.,	46
" Denom. Numb.,	60
" Tables,	44, 49
Definitions, Alphabetical,	195
Denominate Numbers,	51, 55
" " Fractional,	58
" " Decimal,	60
Difference, Common,	160
Discount, Bank,	74
" True,	77
" Commercial,	79
Distances stated,	111
Division,	18–21
" of Com. Frac.,	40
" of Decimals,	49
Divisor, Common,	29
Domestic Bills of Exch.,	86
Doyle's Rule (Lumber),	176
Draft (Exchange),	85
Draft (Duties),	100, 187
Duties,	99, 187
EQUATION of Payments,	100
Equations, Algebraic,	168
Evolution,	122
Exchange.	85–91
Expectation of Life,	180
Exponent, or index,	26
Extraction of Cube Root,	128
" of Square Root,	124
Extremes and Means,	107, 159
FACTORS,	18, 22
Factors, division by,	21
Falling Bodies,	170
French Money,	51
" Exchange,	89
Fractions,	34
Fraction, one number of another,	40

	PAGE
Fractional Den. Numbers,	59
GALLON,	190
Gauging of casks,	176
Geometrical Progression,	161
German Currency,	54
" Exchange,	90
Golden Rule (R. of 3),	109
Government Securities,	185
Gravity or weight,	111, 158, 170
Gravity, Specific,	158
Greatest Common Divisor,	26
HUNDREDTHS,	62
INCLINED PLANE,	171
Income,	91, 92, 93
Insurance,	97
Insurance, Life,	177
Integers,	35
Interest,	65
" found by proportion,	113
" when chargeable,	102
" accurate,	70
" aliquot parts,	68
" by 60 days,	67
" Notes,	80
Inverse Ratio,	114
Invoice,	187
Involution,	122
LETTERS as symbols,	167
Lever,	171
Life Insurance,	177
Longitude, Time,	113
Logs, Measurement of,	175
MANIFEST,	187
Margin,	186
Materials, Strength of,	172
Measure, *i.e.*, divisor,	26
Measurement, Mensuration,	135
" Surfaces,	137, 151
" Volume,	152, 158
Mechanical Powers,	171
Mensuration,	135
" Nos. for reference,	44

	PAGE
Metre, U. S. Coast Survey,	53, 190
Metric System,	52, 190
Mills, when reckoned,	50, 66
Multiplication,	17
" of Decimals,	49
" of Fractions,	39
" Proof of,	27
Multiplier is abstract,	17
NAME or Denominate,	34
Negotiable Note,	74
Nines, Casting out,	27
Notation,	9–12
Note, Promissory,	74
" Face of	77
" Proceeds of	75
Number, Definition,	25
Numbers, classified,	25
" Abstract,	9
" Concrete,	9
" Denominate,	55
Numeration,	9–12
OBJECT Lessons,	126, 136
"Of" precedes the divisor,	40
Orders,	10
Overs,	27
PAR of Exchange,	87
Parallelopiped,	152
"Part," as one No. of another,	40
Partial Payments,	83
Parenthesis,	18
Penalties for Usury,	184
Pendulum,	170
Percentage,	61, 62
Percent. Table,	62
Periods,	10
Personal Property,	98
Policy,	97, 177
Poll Tax,	98
Polygons,	140
Polyhedrons,	152
Pounds, kinds and values,	190
Powers,	122
Preface,	5

	PAGE
Premium,	97, 177
Present Worth,	77, 183
Prime Numbers,	26
Principal,	61
Problem Tables,	16
Problems, Mech. Powers,	168
Proceeds of Note,	75
Profit and Loss,	93
Progression, Arithmetical,	159
" Geometrical	167
Proof, Strict,	27
" Importance of,	29
" of Division,	29
Properties of Numbers,	25
Proportion,	107, 114
" of similar figures,	109
" of similar solids,	133, 158
Puts, Stockbroker's,	186
Pulley,	171
Pyramid,	134
QUOTATION (exchange),	88, 90
RADICAL sign,	124, 168
Railroad Freights,	189
Rate Per Cent.,	61, 64, 71
Ratio,	106, 161
" Direct,	110
" Inverse,	114
Real Property,	98
Reciprocal,	107
Recurring Decima's,	47
Reduction,	13
" D. N.,	55
" Frac. D. N.,	58
" Fractions,	36–46
Reichmarks (Marks),	89
Rentals,	93
Reversionary Annuity,	181
Review of Addition—Divis'n,	31
Right Ang. Triang.,	127
Rings, Surface, etc.,	149
Root, Cube,	128
" Square,	124
Rule of Three,	109
SAWED Lumber, Rule,	176

	PAGE
Scrip,	185
Sector,	145
Segment,	145, 153
Screw,	171
Series,	159, 161, 162
Signs, letters, etc.,	24, 167
Slate Ex., Drill,	8
Solids compared,	133, 158
" Volume of	158
Solution defined,	13
Specific Duties,	187
Specific Gravity,	111, 174
Squares, Perfect,	25, 26, 123
Square and Circle compared,	148
Square Root,	124
STANDARD Unit,	51
Statement of Probl's,	7, 38, 41, 110
Statute of Limitations,	184
Sterling Exchange,	88
Stocks and Bonds,	91, 185
Stockbrokers' Terms,	186
Storage,	188
Stone Masonry,	175
Subtraction,	16
" of Decimals,	48
" of Fractions,	39
Suggestions to Teachers,	7
Surrender Value,	178
TABLE, Fundamental,	51
" of Decimal Notation,	44
" for Decimal Problems,	49
" Miscell. "	15
" of Germ. Currency,	54
" of Metric System,	52
" of Notation and Num.,	11
" of Powers,	123
" of Prime Numbers,	26
" of Tensile Strength,	173
" of Transverse "	173
Tables of Weights, etc.,	51, 54, 174
Tare,	187
Tariff,	187
Taxes,	98
Time,	71, 113, 185
Trapezoids,	140
Triangles, Right Ang.,	127
True Discount,	77
UNIT,	9, 51
U. S. Currency,	50
Usury,	76
VOCABULARY, Alphab.,	195
Volume,	135
WATER,	111
Weight of dif. Substances,	111
Wheel and Axle,	171